Analysis and Design of Planar Self−Balancing Double−Pendulum Robots

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What makes a robot good at balancing?

- when a relatively small physical response can correct a relatively large disturbance

What are the limiting factors?

- the effectiveness of the control system
- the quality of the sensors
- the speed and strength of the actuators
- physical properties of the robot mechanism: kinematics and mass distribution

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How can we measure the balancing performance of a robot mechanism?

- the ratio of physical effort (movement of actuated joints) to result (movement of centre of mass relative to support)

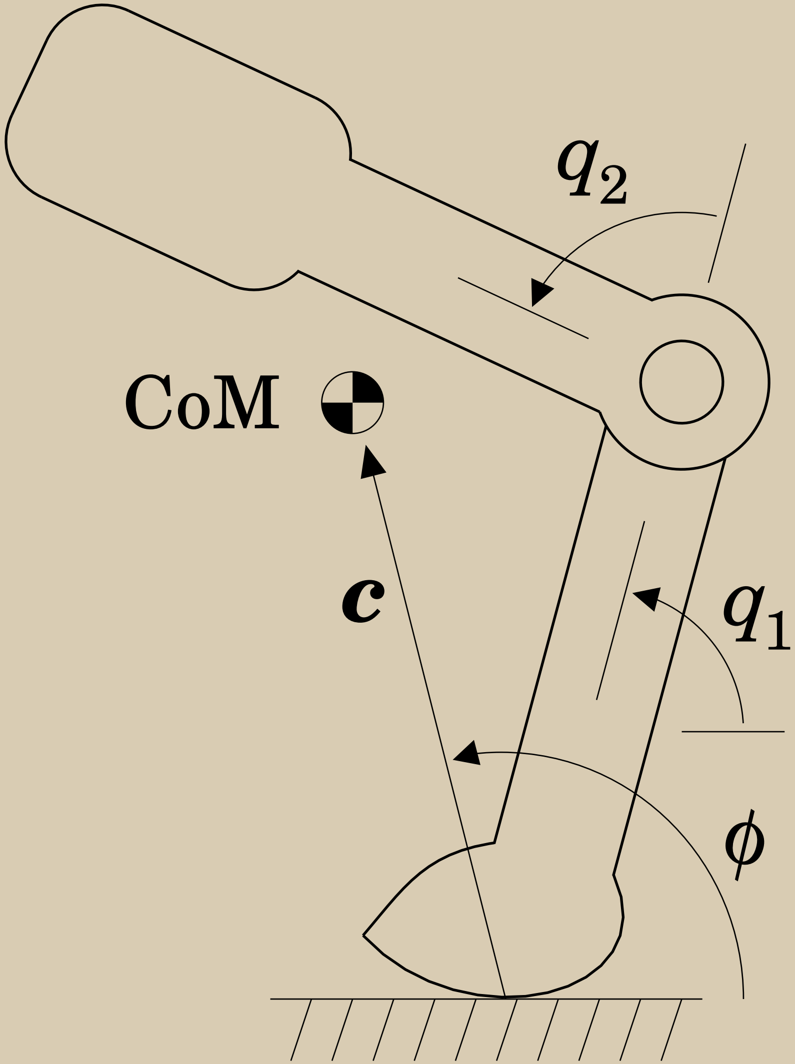
This talk describes one such ratio: the *velocity gain*.

Why are such measures useful?

By defining quantitative measures of a robot mechanism's physical capacity to balance, we obtain a tool to

- analyse the performance of existing mechanisms
- design new mechanisms to achieve a specified performance
- guide the development of control systems for balancing

Velocity Gain (for a planar double pendulum)



Definition:

$$G_{\mathrm{v}} = \frac{\Delta\dot{\phi}}{\Delta\dot{q}_2}$$

$\Delta\dot{\phi}$ ⟵ result

$\Delta\dot{q}_2$ ⟵ effort

in response to an impulse at joint 2

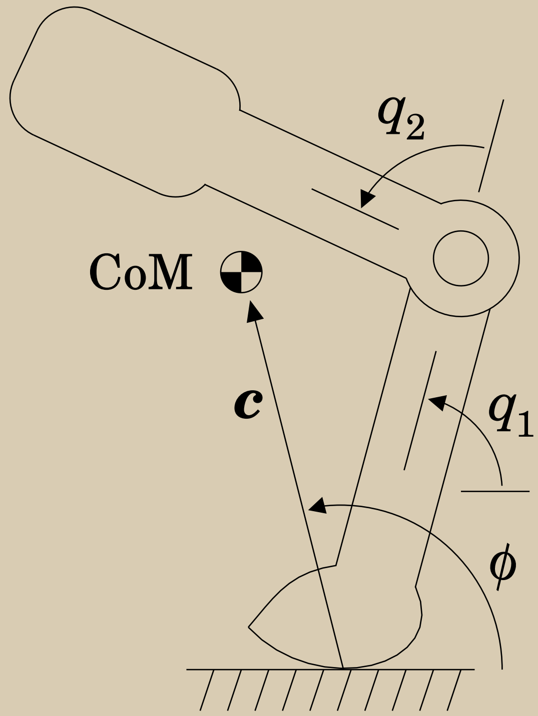
Calculation

Let u be an impulse applied at joint 2. The equation of impulsive motion is

$$\begin{bmatrix} \Delta\dot{q}_1 \\ \Delta\dot{q}_2 \end{bmatrix} = \boldsymbol{H}^{-1} \begin{bmatrix} 0 \\ u \end{bmatrix}$$

which can be solved to give

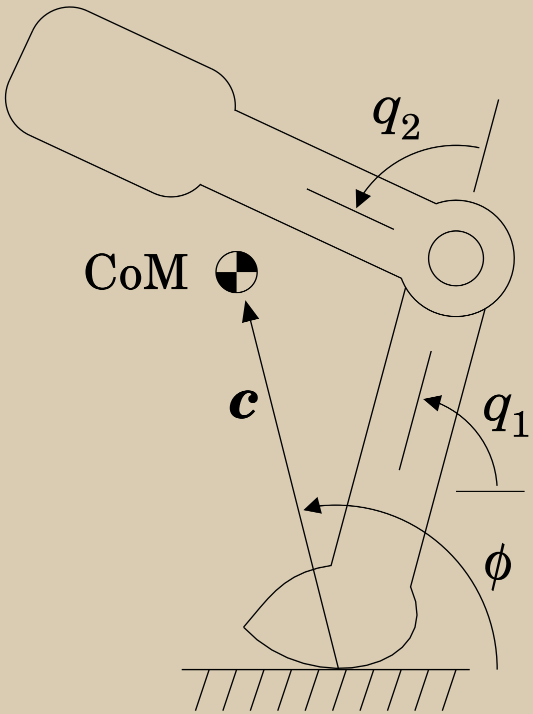
$$\frac{\Delta\dot{q}_1}{\Delta\dot{q}_2} = \frac{-H_{12}}{H_{11}}$$



Calculation

$\boldsymbol{c}$ is a function of q_1 and q_2, so

$$\dot{\boldsymbol{c}} = \frac{\partial \boldsymbol{c}}{\partial q_1}\dot{q}_1 + \frac{\partial \boldsymbol{c}}{\partial q_2}\dot{q}_2$$



therefore

$$\frac{\Delta\dot{\boldsymbol{c}}}{\Delta\dot{q}_2} = \frac{\partial \boldsymbol{c}}{\partial q_1}\frac{\Delta\dot{q}_1}{\Delta\dot{q}_2} + \frac{\partial \boldsymbol{c}}{\partial q_2} = \frac{\partial \boldsymbol{c}}{\partial q_2} - \frac{\partial \boldsymbol{c}}{\partial q_1}\frac{H_{12}}{H_{11}}$$

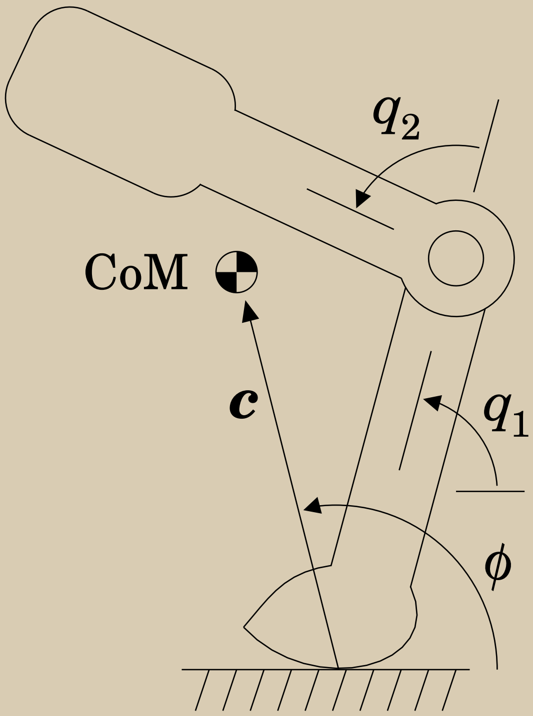
Calculation

And finally,

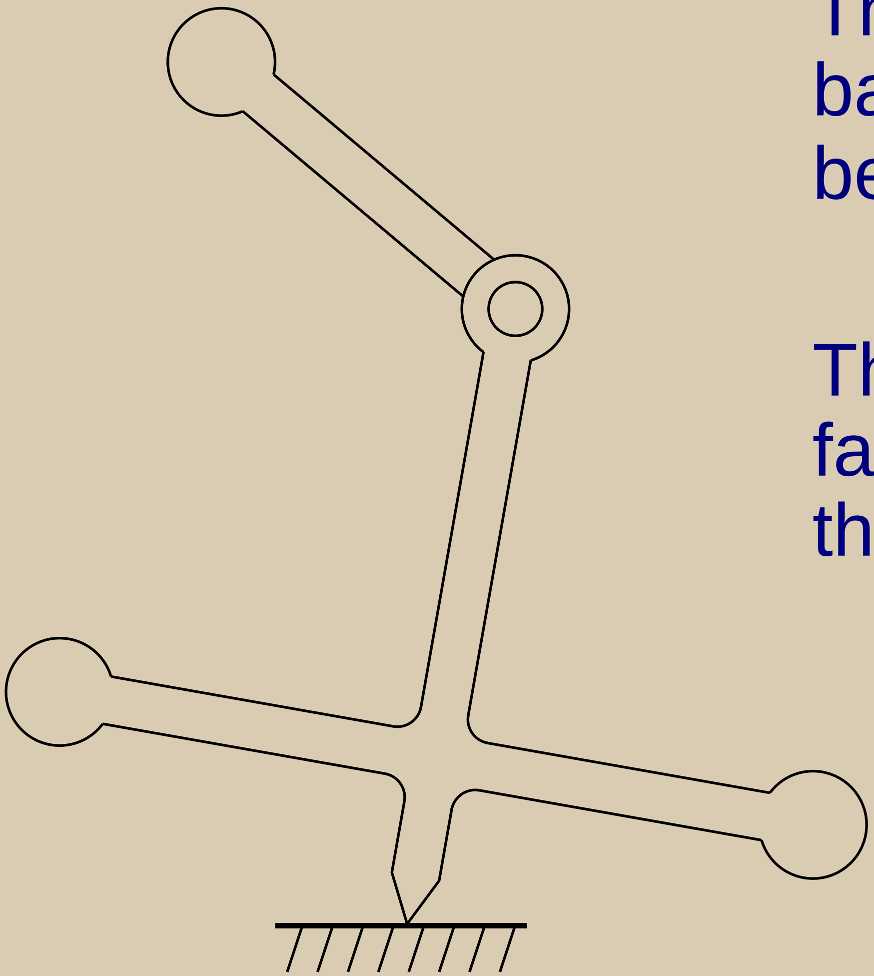
$$\dot{\phi} = \frac{\boldsymbol{b} \cdot \dot{\boldsymbol{c}}}{|\boldsymbol{c}|}$$

where $\boldsymbol{b}$ is a unit vector perpendicular to $\boldsymbol{c}$ in the direction of increasing ϕ. So

$$G_{\mathrm{v}}(\boldsymbol{q}) = \frac{\Delta\dot{\phi}}{\Delta\dot{q}_2} = \frac{\boldsymbol{b} \cdot \Delta\dot{\boldsymbol{c}}}{|\boldsymbol{c}|\,\Delta\dot{q}_2}$$



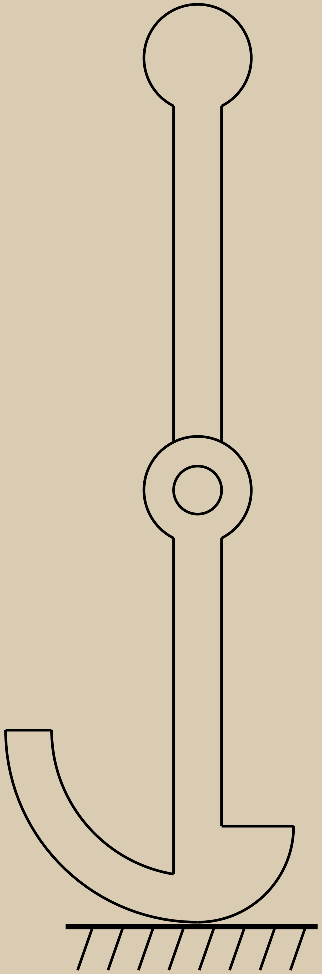
Example 1



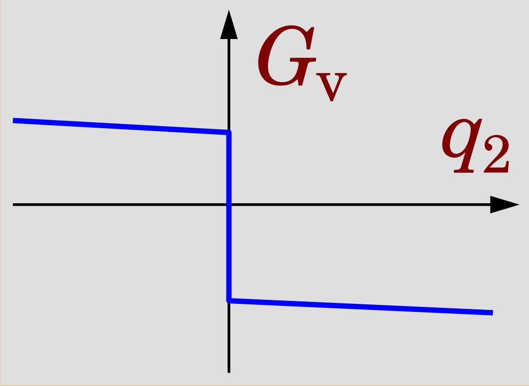
This mechanism cannot balance in any configuration because $G_{\mathbf{v}} = 0$ everywhere.

There is a four−parameter family of mechanisms with this property.

Example 2

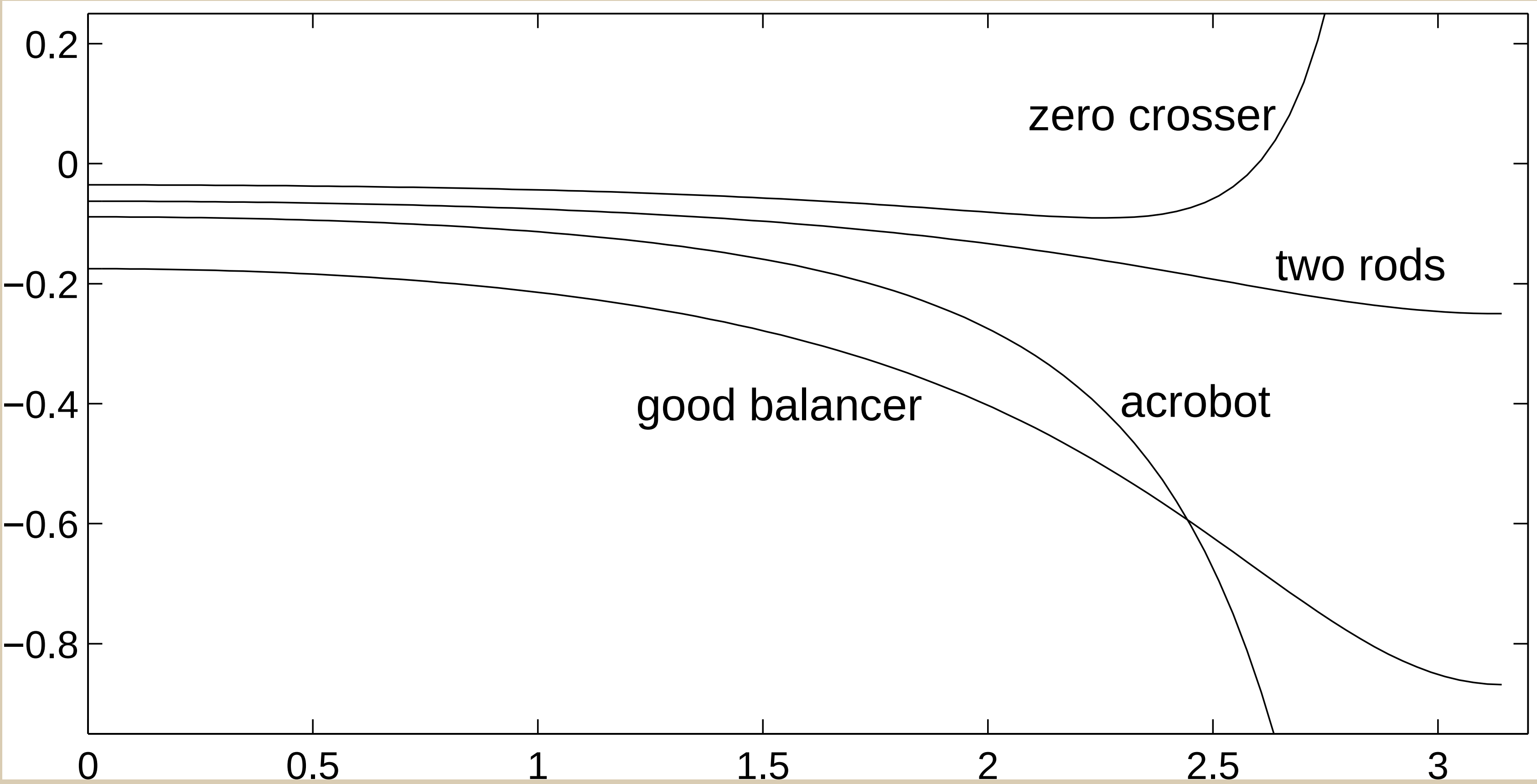


This mechanism cannot balance in this configuration because G_{v} crosses zero.



Balancing at nearby configurations is risky.

Some More Examples



Extensions

The concept of velocity gain can be extended to:

- mechanisms that balance in 3D
- mechanisms with more than two main bodies

Current Work

We are currently using the velocity gain to:

- design a 3D double−pendulum balancer
- design a 3D hopping machine
- guide the design of balancing algorithms in 2D and 3D

Conclusion

Velocity gain is a measure of a robot mechanism's intrinsic ability to balance. It can be used to

- analyse existing mechanisms
- design new mechanisms to meet a given performance target
- guide the development of control systems for balancing
- explore the problem of balancing