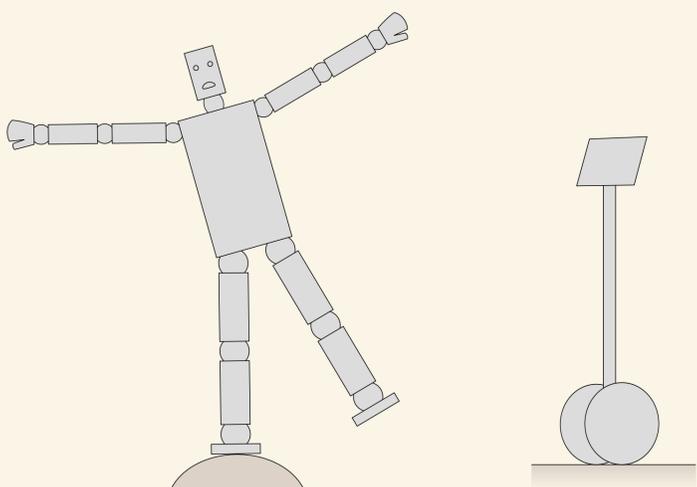


# Quantitative Measures of a Robot's Ability to Balance

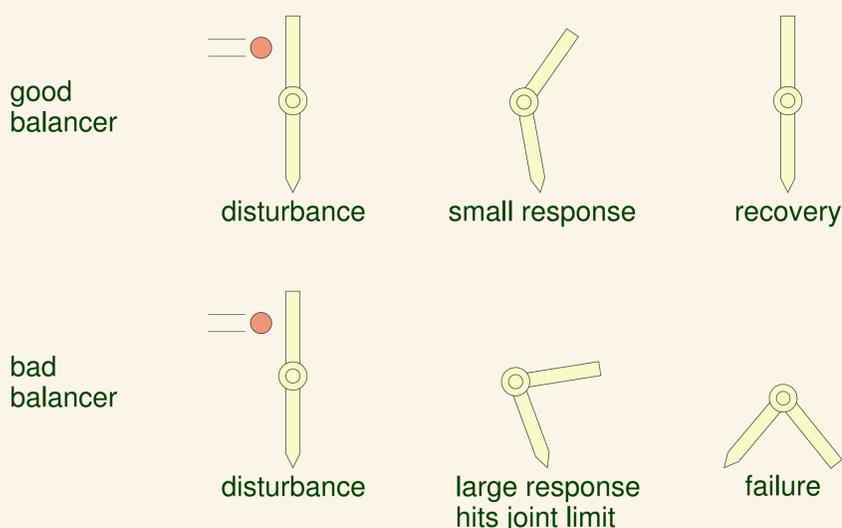
## The Problem



How good are these robots at balancing? How big a disturbance will make them fall over? To answer questions like this, we need *quantitative measures* of a robot's *physical ability to balance*.

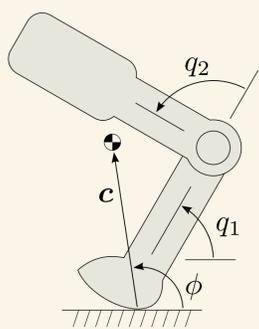
## The Solution

A robot is good at balancing if a relatively small movement of the actuated joint(s) is enough to correct a balance error.



So we define the measure as the ratio of the amount of movement of the centre of mass to the amount of joint motion needed to produce it.

In particular, we define the robot's *velocity gain* as the ratio of a change in centre of mass velocity to a change in joint velocity, both changes being produced by an impulse from the actuator.



For a planar double pendulum, we can define an *angular velocity gain* as

$$G_\omega = \frac{\Delta \dot{\phi}}{\Delta \dot{q}_2}$$

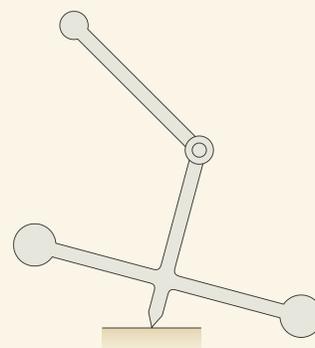
and a *linear velocity gain* as

$$G_v = \frac{\Delta \dot{c}_x}{\Delta \dot{q}_2}$$

where  $\Delta \dot{\phi}$ ,  $\Delta \dot{c}_x$  and  $\Delta \dot{q}_2$  are step changes in the velocities  $\dot{\phi}$ ,  $\dot{c}_x$  and  $\dot{q}_2$  caused by a nonzero impulse at joint 2.

More general definitions are presented in the paper.

## Examples

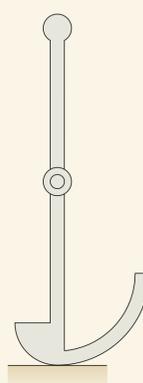


### The 'impossible' balancer

(from Featherstone 2013)

It is physically impossible to balance this mechanism because  $G_\omega = 0$  in every configuration.

There are infinitely many mechanisms with this property.



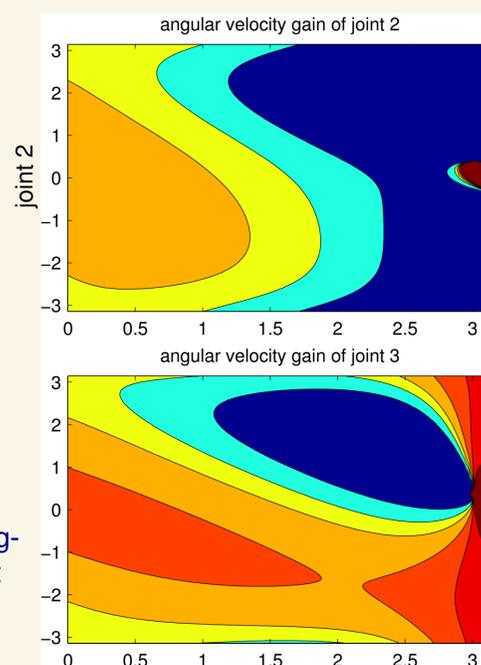
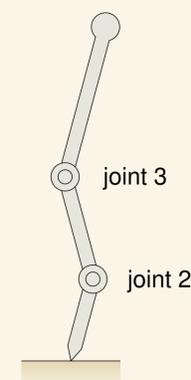
### A foot shape problem

(from Featherstone 2013)

This mechanism has a foot composed of two circular arcs. There is a step-change in curvature where the two arcs meet.  $G_\omega > 0$  along one arc and  $G_\omega < 0$  along the other.

The robot cannot balance at the transition point (shown) because it has two ways to correct a balance error in one direction and no way to correct a balance error in the other.

## Velocity Gain Maps



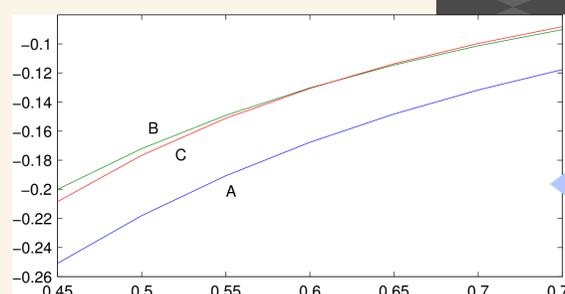
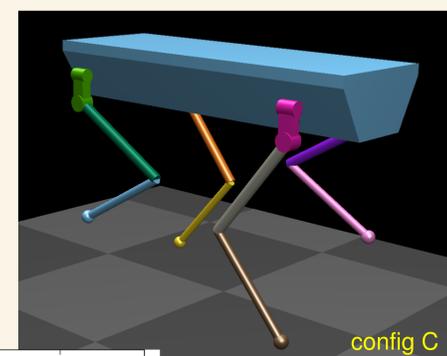
Plots of velocity gain vs. configuration reveal where the robot is good and bad at balancing, and which joints to use.

## Analysis of Existing Robots

How well can HyQ balance on two feet?

What are the best configurations for balancing?

What is a good movement for balancing?



$G_\omega$  vs. torso height for three configurations of the free legs