

# High Performance Balancing on a Narrow Support

by Roy Featherstone

presented at the ICRA 2021 Workshop on  
High Dynamic Motion Generation  
for Under-Actuated Robots



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First view this video:

EITHER

download the shared file [icra2021rf.mp4](#) (if it exists) from the zoom chat panel (if you are using zoom) and view it

OR

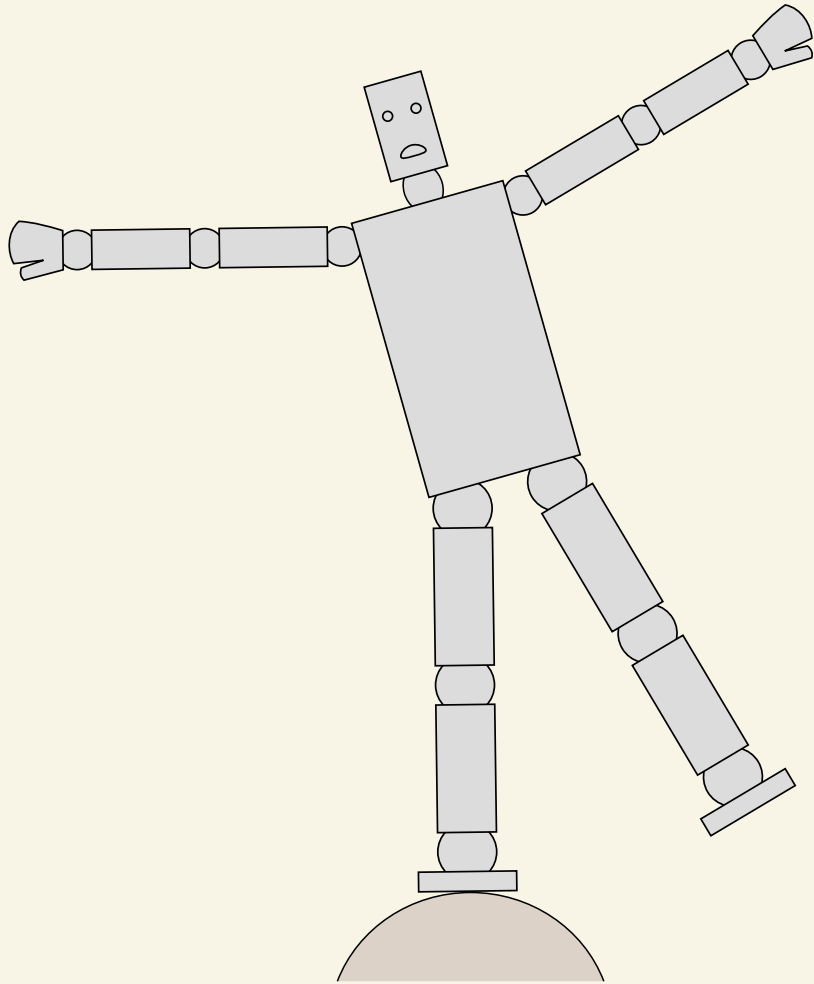
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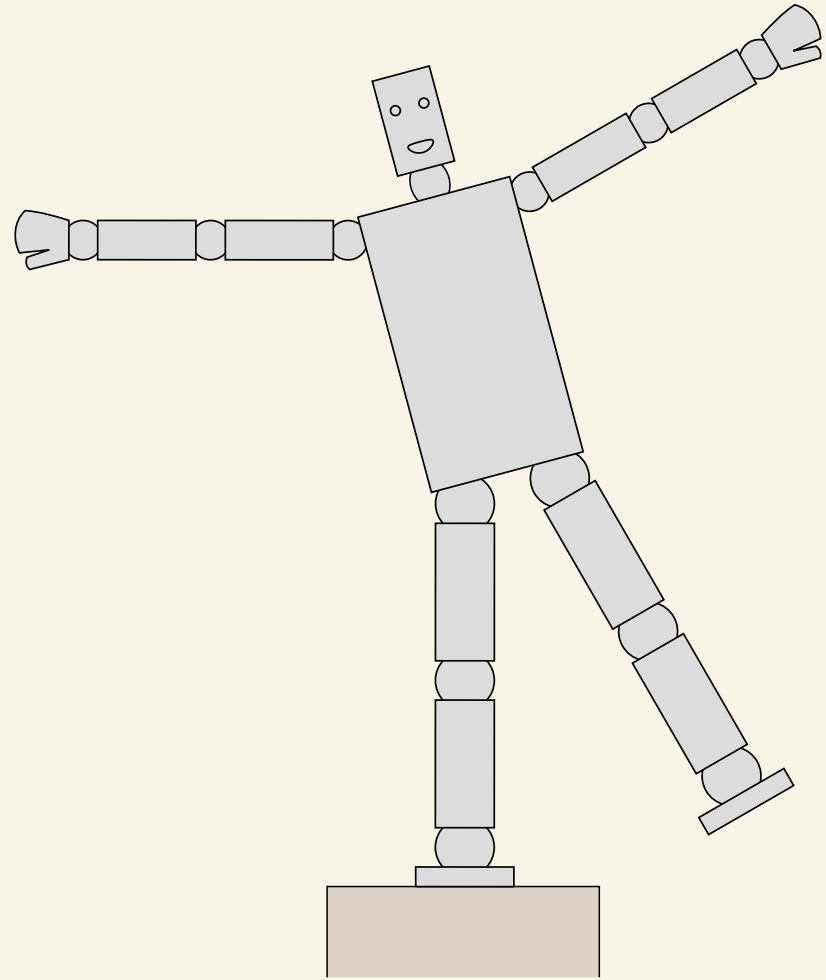
Then return to this slide show.

Waiting for people to return.

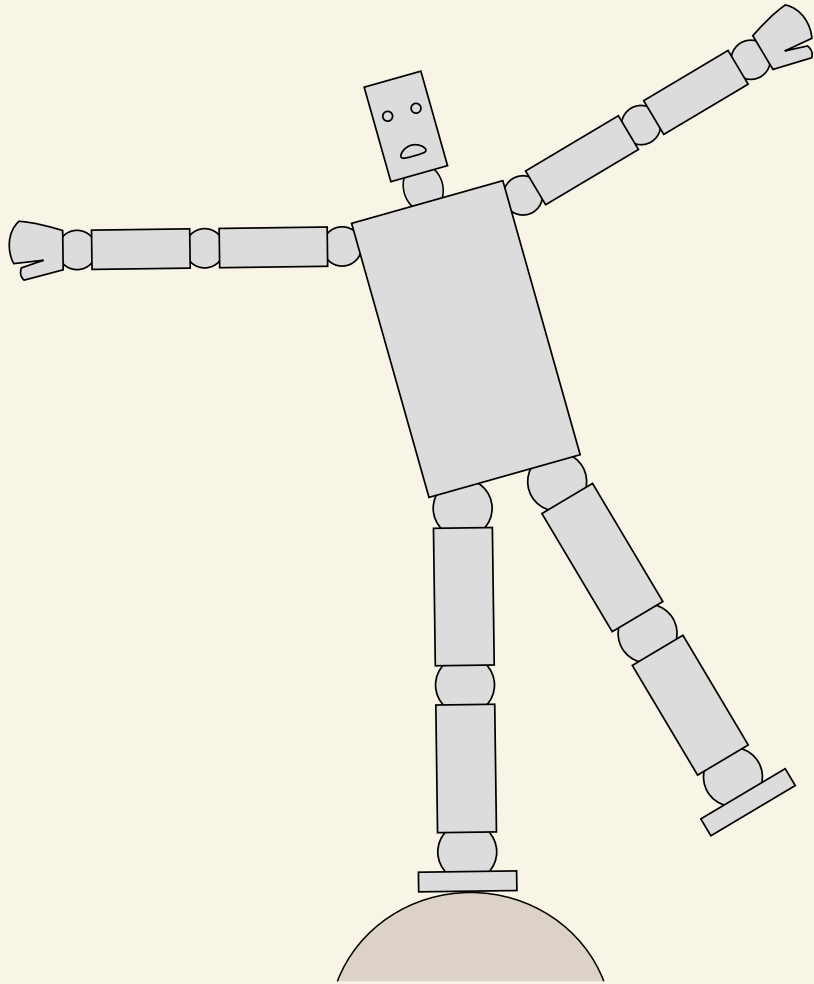
The talk will continue soon.



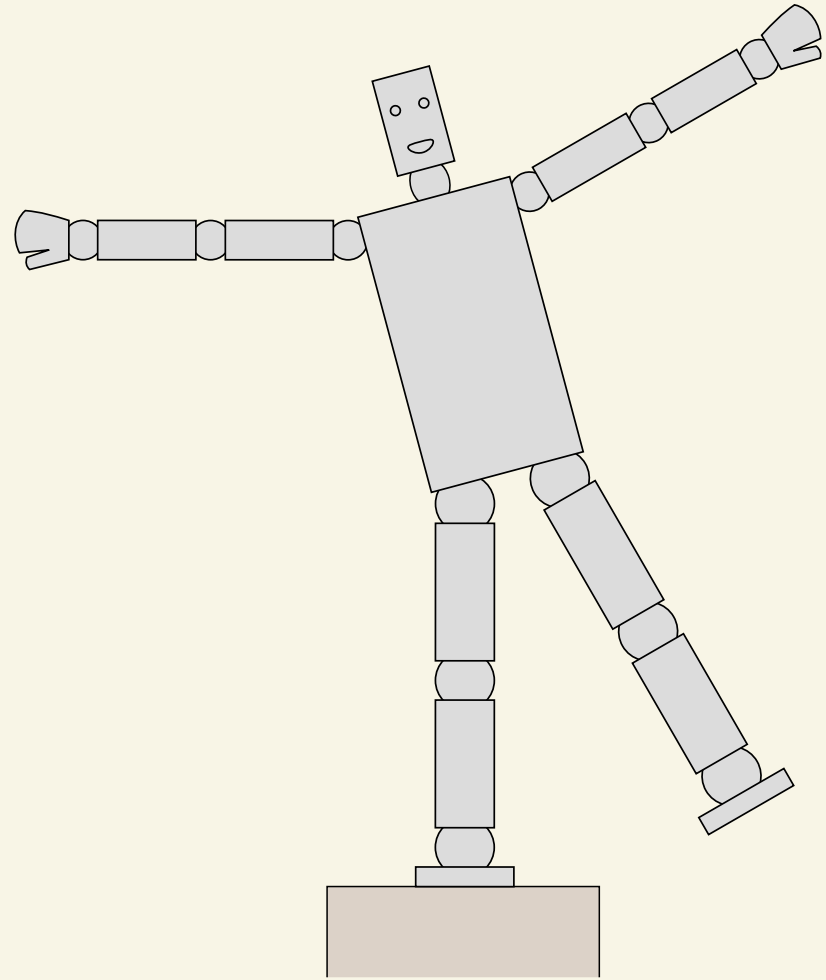
narrow support —  
effectively a single point



substantial polygon  
of support



dynamically unstable —  
must balance actively



statically stable as long as  
ground reaction force stays  
inside support polygon

# High-Performance Balancing

## Definition:

A robot is a high-performance balancer if it can

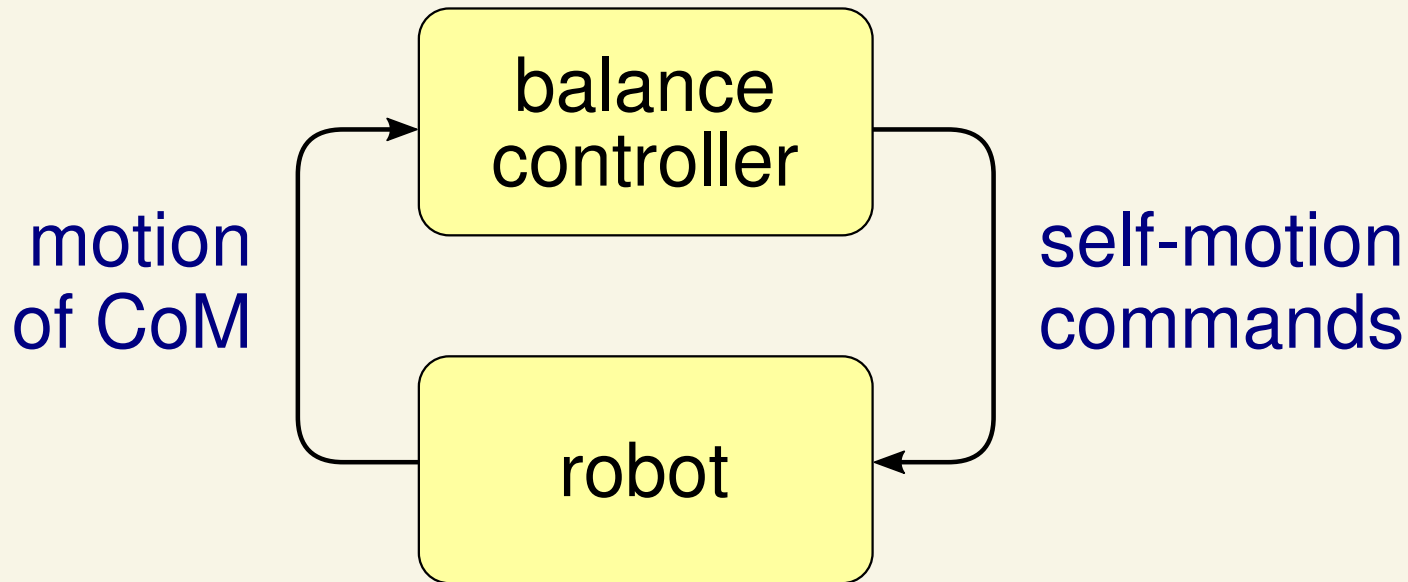
- accurately follow commands to make large, fast movements without losing its balance; and
- quickly recover from large balance disturbances

# High-Performance Balancing

## Our Approach:

1. Balancing is a physical activity, not just a control theory exercise;
  - so study the *physics of balancing*
2. A robot's physical ability to balance is a property of the robot itself, not the control system;
  - so study what makes a robot good at balancing, and make your robot *good by design*.

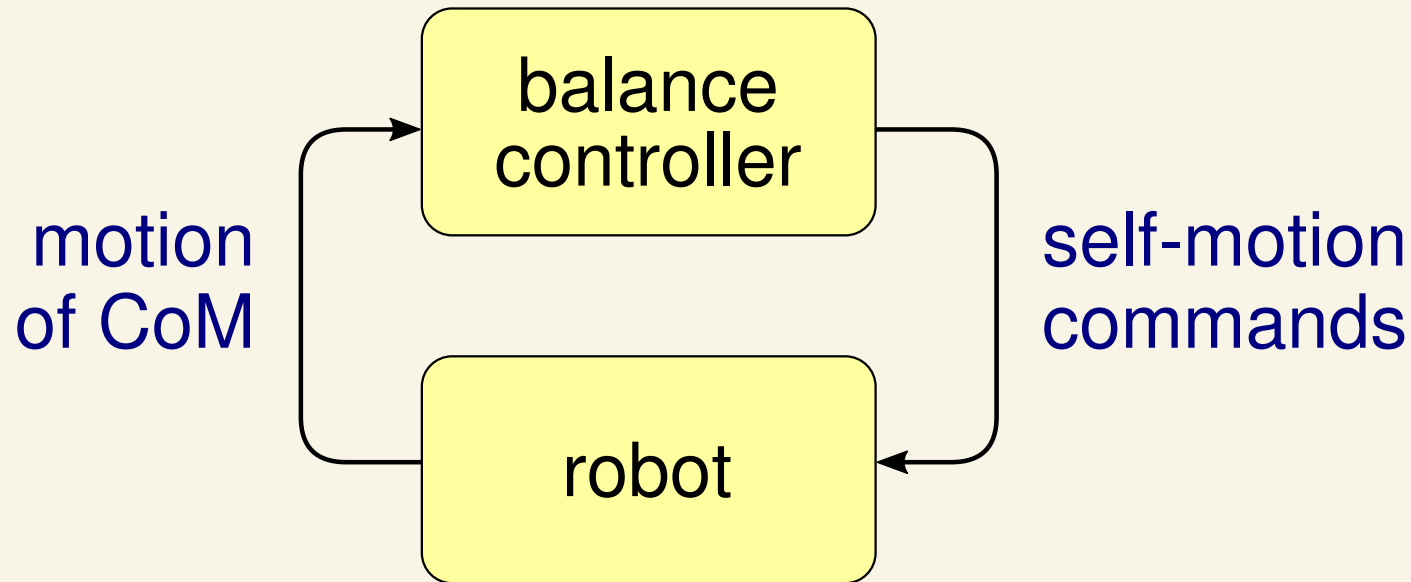
# Physical Ability to Balance on a Point



Balancing on a point requires moving the robot's centre of mass (CoM) relative to the support point by means of self motions (i.e., motions of the actuated joints).



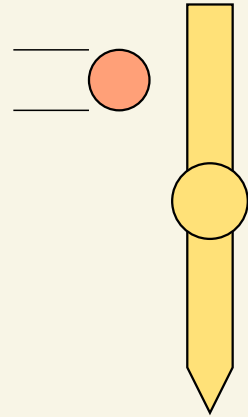
# Physical Ability to Balance on a Point



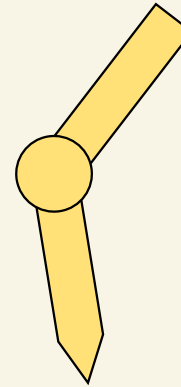
So a robot is *physically good at balancing* if a relatively small self motion causes a relatively large CoM motion.

# Physical Ability to Balance on a Point

good balancer



disturbance

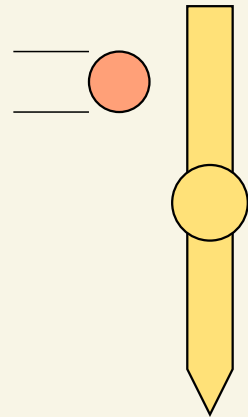


response

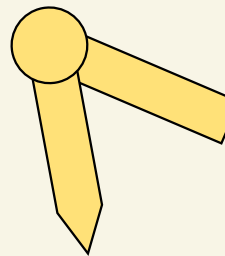


recovery

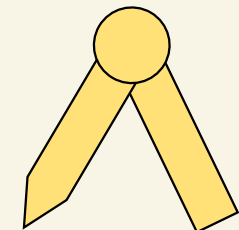
bad balancer



disturbance



response hits  
motion limit



fall

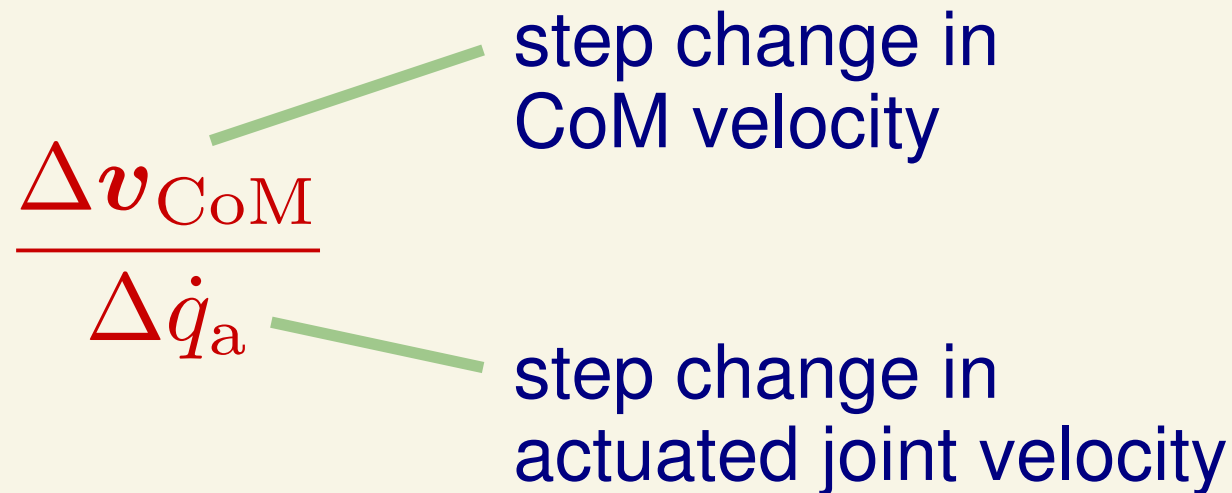
# Physical Ability to Balance on a Point

This leads to the idea of *velocity gain*:

$$\frac{\Delta \mathbf{v}_{\text{CoM}}}{\Delta \dot{q}_a}$$

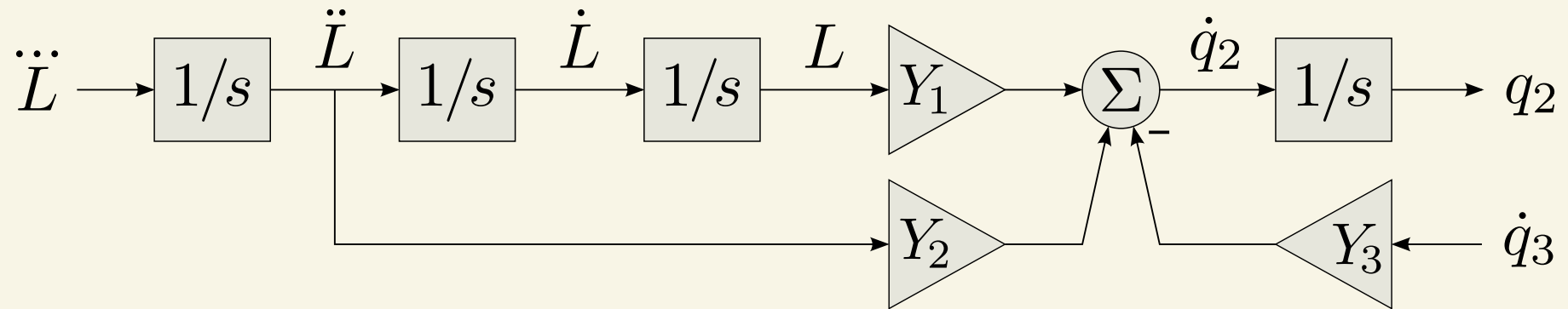
step change in CoM velocity

step change in actuated joint velocity



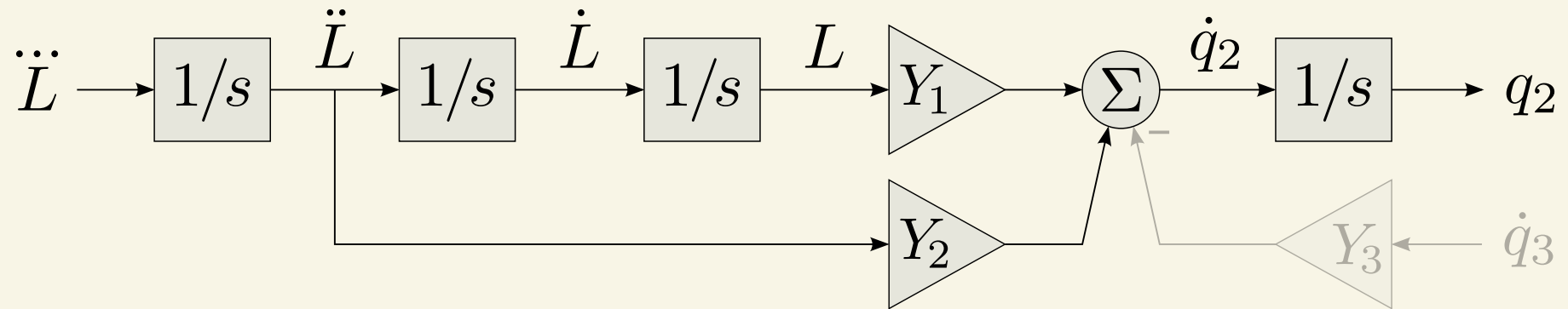
both caused by an impulse at the actuated joint

# The Simple Dynamics of Balancing



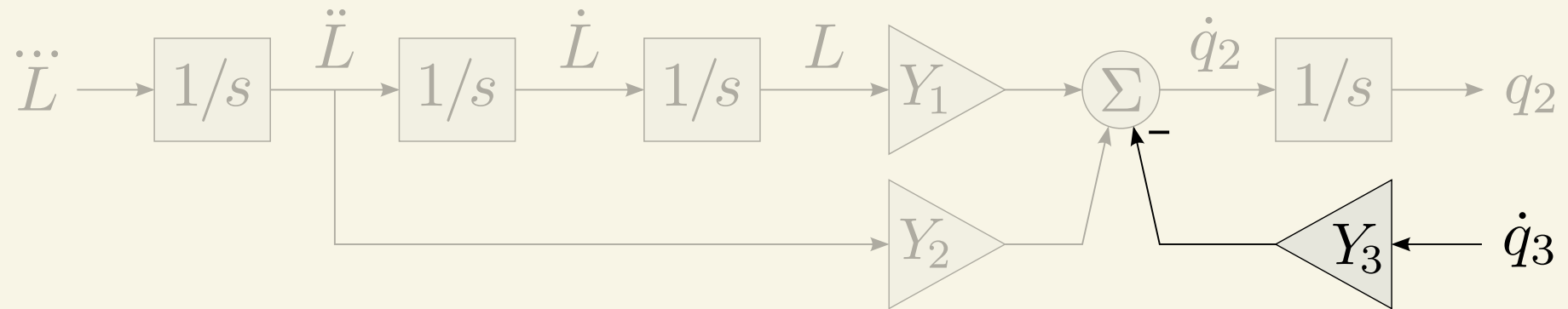
The physical process of balancing can be expressed in the form of this block diagram, which serves as the *plant* for the balance controller to control.

# The Simple Dynamics of Balancing



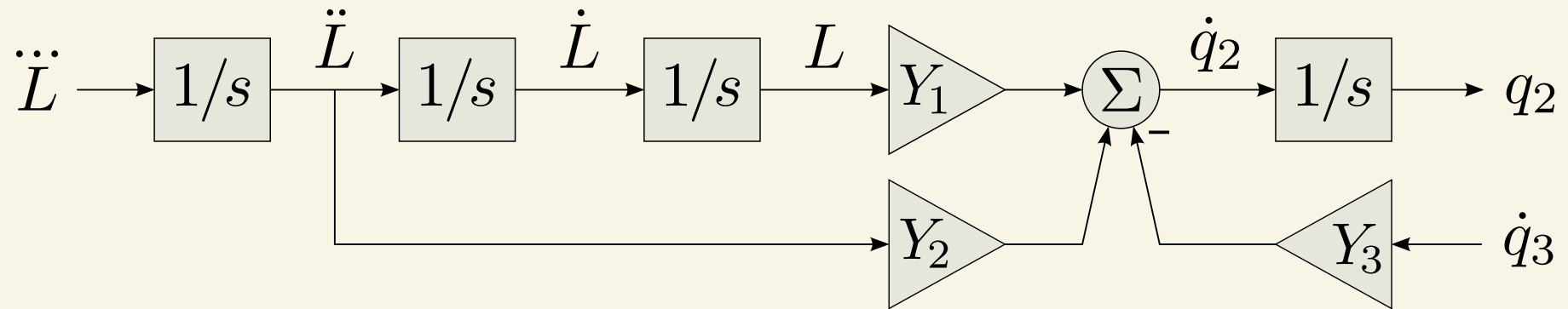
This portion is concerned with balancing

# The Simple Dynamics of Balancing



and this portion describes  
the disturbances caused by  
other motions.

# The Simple Dynamics of Balancing



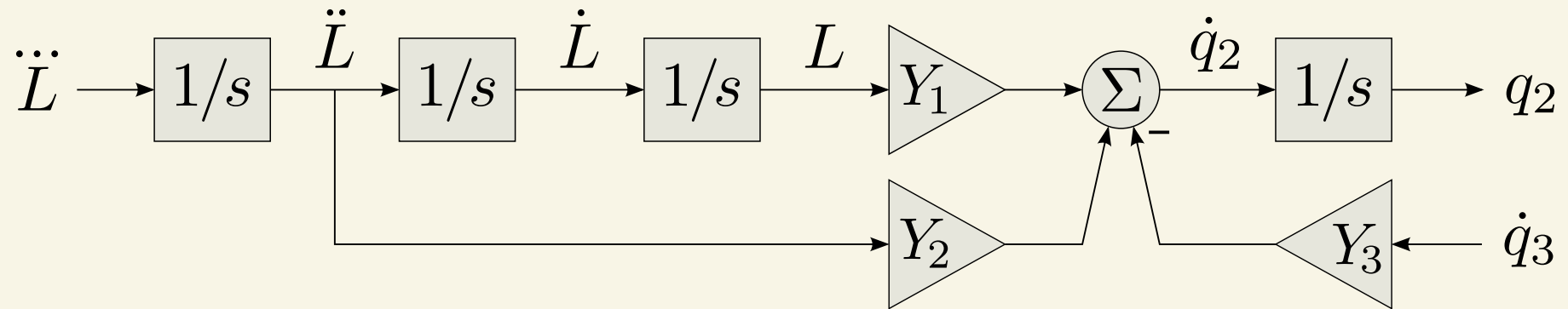
$L$  angular momentum of the robot about the support

$q_1$  variable(s) describing the robot's motion relative to the ground

$q_2$  *overloaded* variable(s) used *both* to balance *and* to follow motion commands

$q_3$  all other robot variables

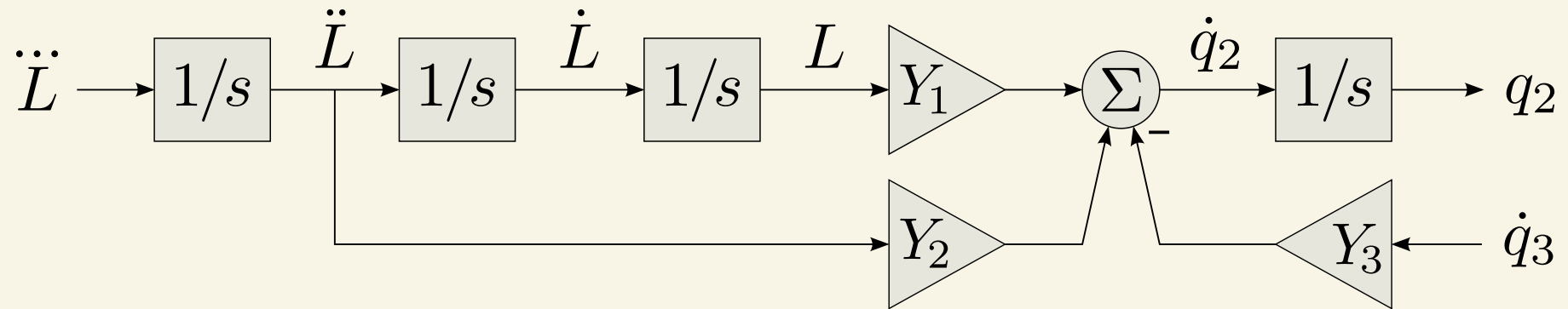
# The Simple Dynamics of Balancing



The balancing behaviour of the robot depends only on the two gains  $Y_1$  and  $Y_2$  which are easily calculated functions of the robot's configuration variables.



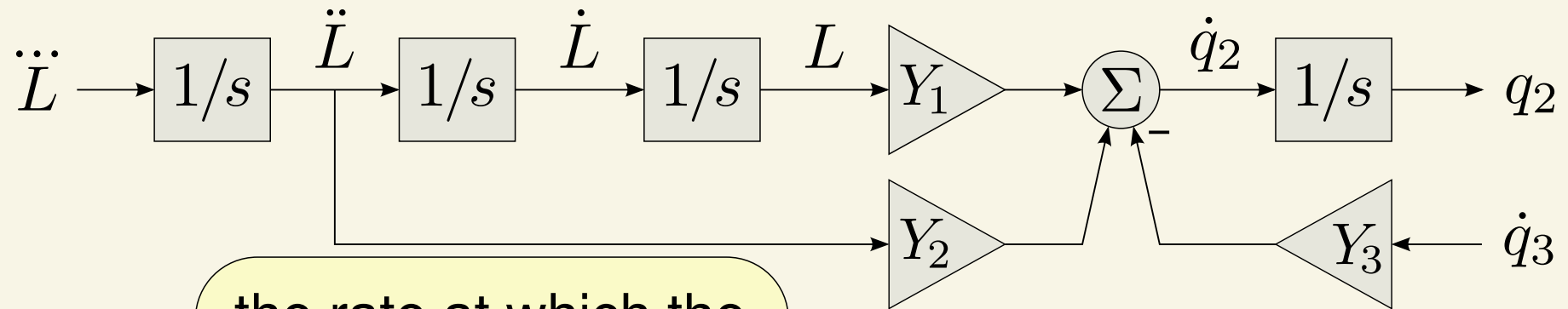
# The Simple Dynamics of Balancing



$Y_1$  and  $Y_2$  are functions of two easily measured physical properties of the robot:

- the natural time constant of toppling, and
- the linear velocity gain of  $\dot{q}_2$

# The Simple Dynamics of Balancing



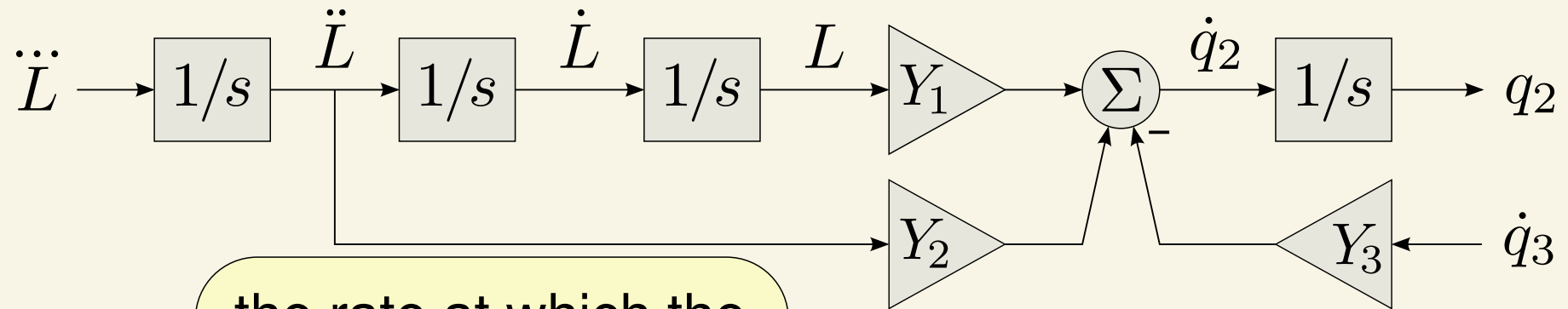
$Y_1$  and  
properties

the rate at which the robot begins to fall, treating it as a single rigid body

two easily measured physical

- the natural time constant of toppling, and
- the linear velocity gain of  $\dot{q}_2$

# The Simple Dynamics of Balancing



$Y_1$  and  
properties

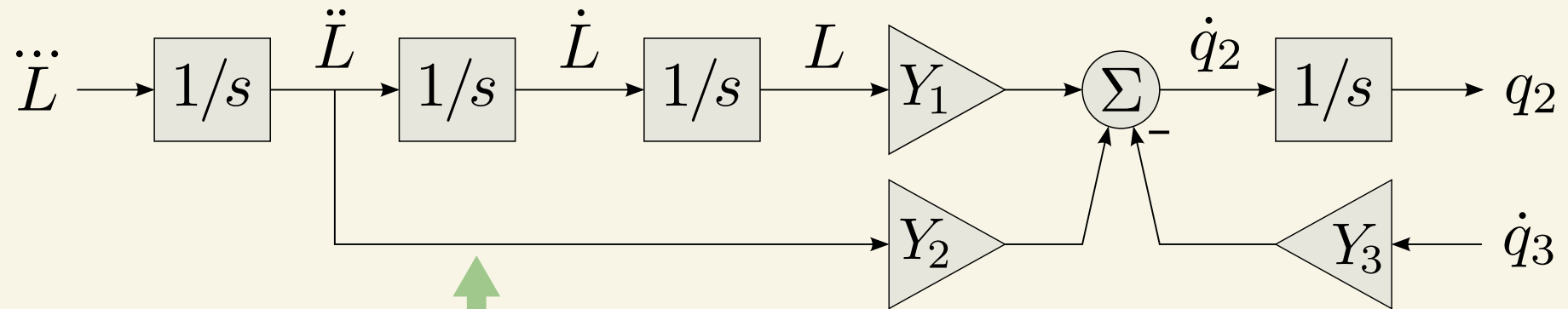
the rate at which the robot begins to fall, treating it as a single rigid body

two easily measured physical

- the natural time constant of toppling and
- the linear velocity gain of  $\dot{q}_2$

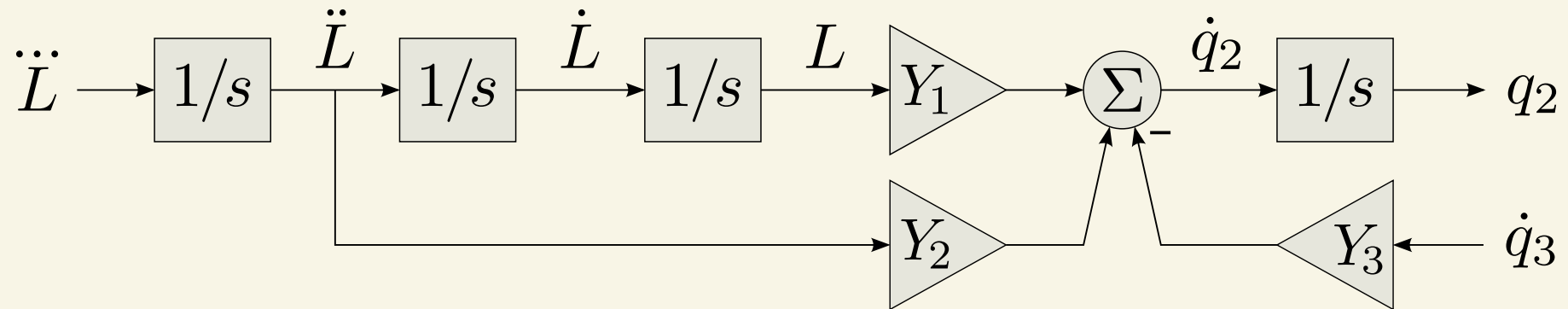
the step change in CoM velocity caused by a unit step change in  $\dot{q}_2$

# The Simple Dynamics of Balancing



In 2D, these signals are all scalars.  
In 3D, they are 2D vectors.

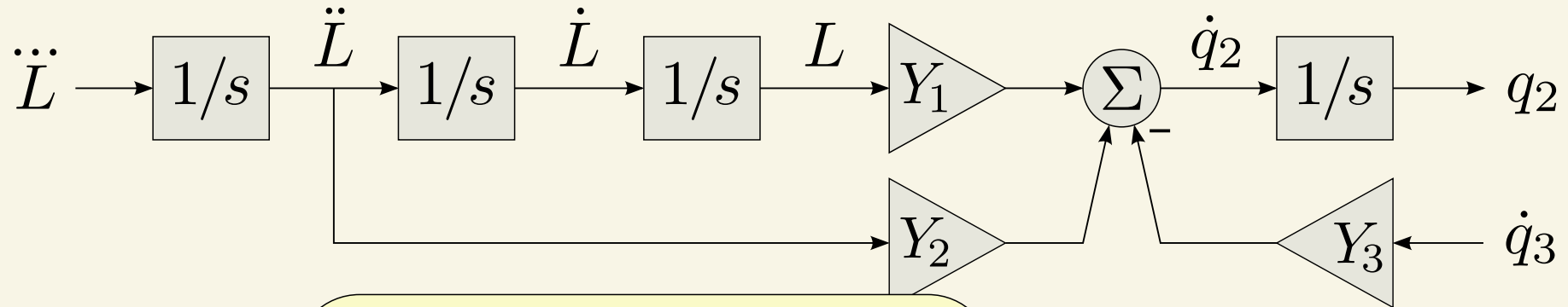
# Simple Balance Control



High-performance balancing can be achieved using

- a simple control law closed around *this plant*
- and a simple *acausal filter* that implements *leaning in anticipation* of the balance disturbances that will be caused by commanded future motions.

# Simple Balance Control



High-performance

a first-order low-pass filter running *backwards* in time from the future to the present

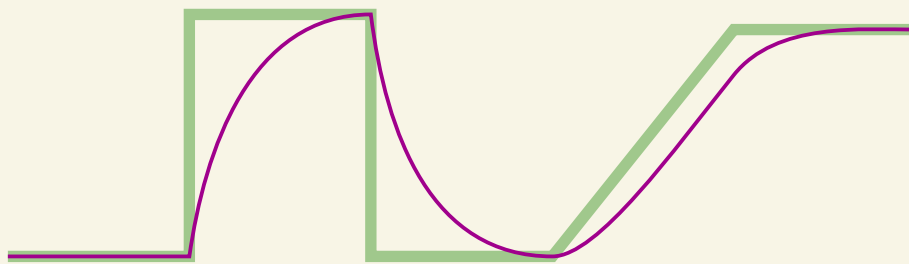
achieved using

- a simple *and this plant*
- and a simple *acausal filter* that implements *leaning in anticipation* of the balance disturbances that will be caused by commanded future motions.

# The Acausal Filter

a first-order low-pass filter running backwards in time to cancel the non-minimum-phase response of the robot

normal low-pass filter

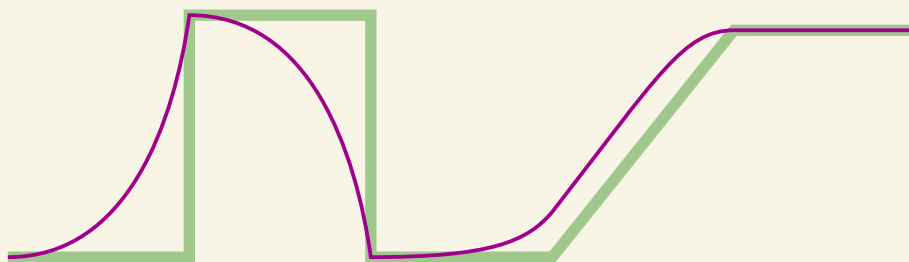


transfer function

$$\frac{1}{1 + T_c s}$$

natural time constant of toppling

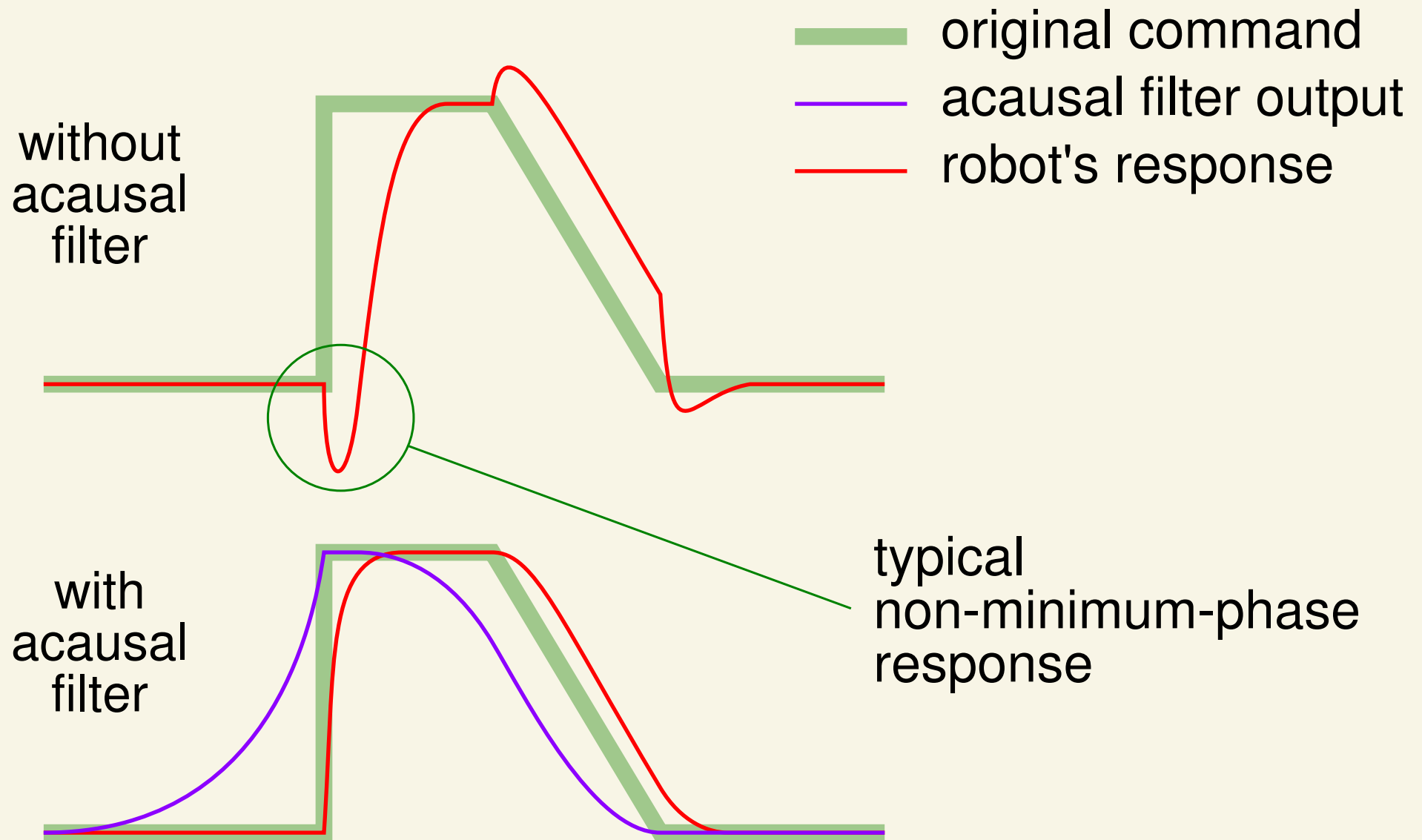
acausal low-pass filter



$$\frac{1}{1 - T_c s}$$

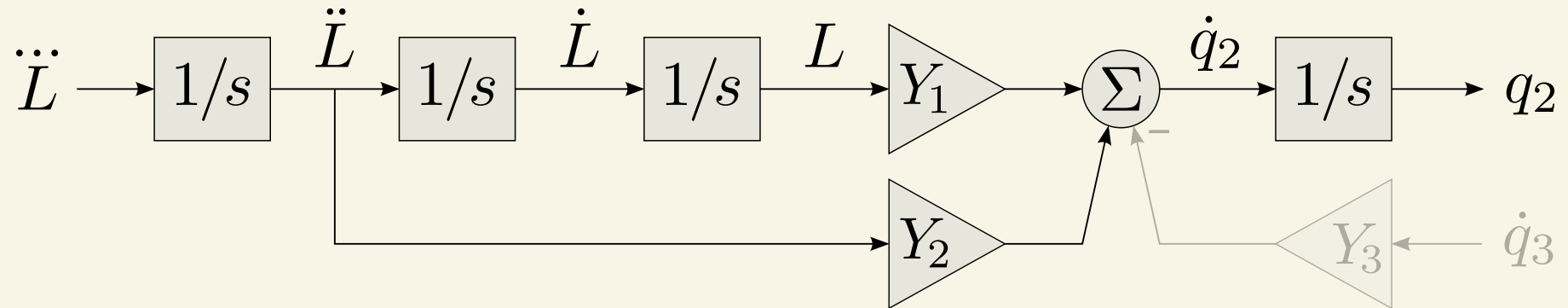
unstable pole in forward time

# The Acausal Filter



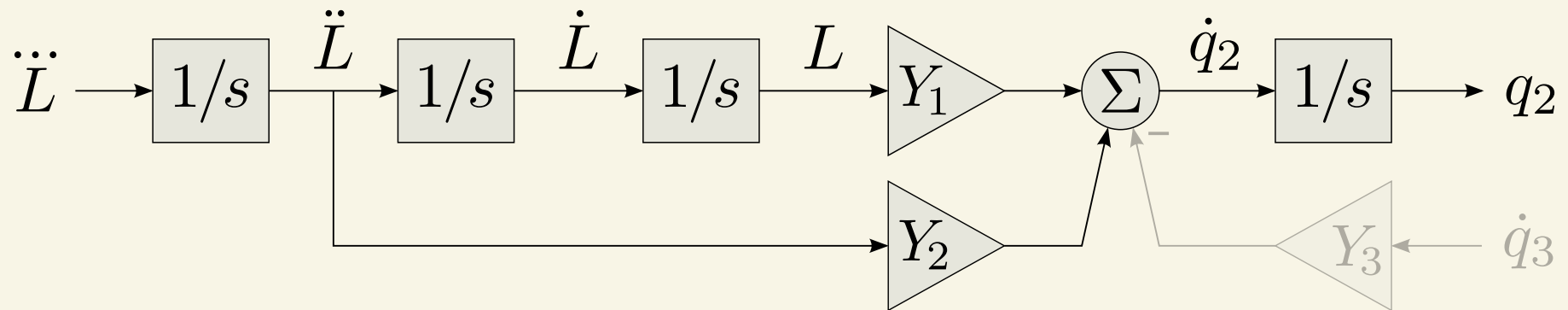


# Balance Control Law



$$\ddot{L} = k_{dd}\ddot{L} + k_d\dot{L} + k_L L + k_q(q_2 - AF(u))$$

# Balance Control Law



$$\ddot{L} = \underline{k_{dd}} \ddot{L} + \underline{k_d} \dot{L} + \underline{k_L} L + \underline{k_q} (q_2 - AF(u))$$

feedback gains

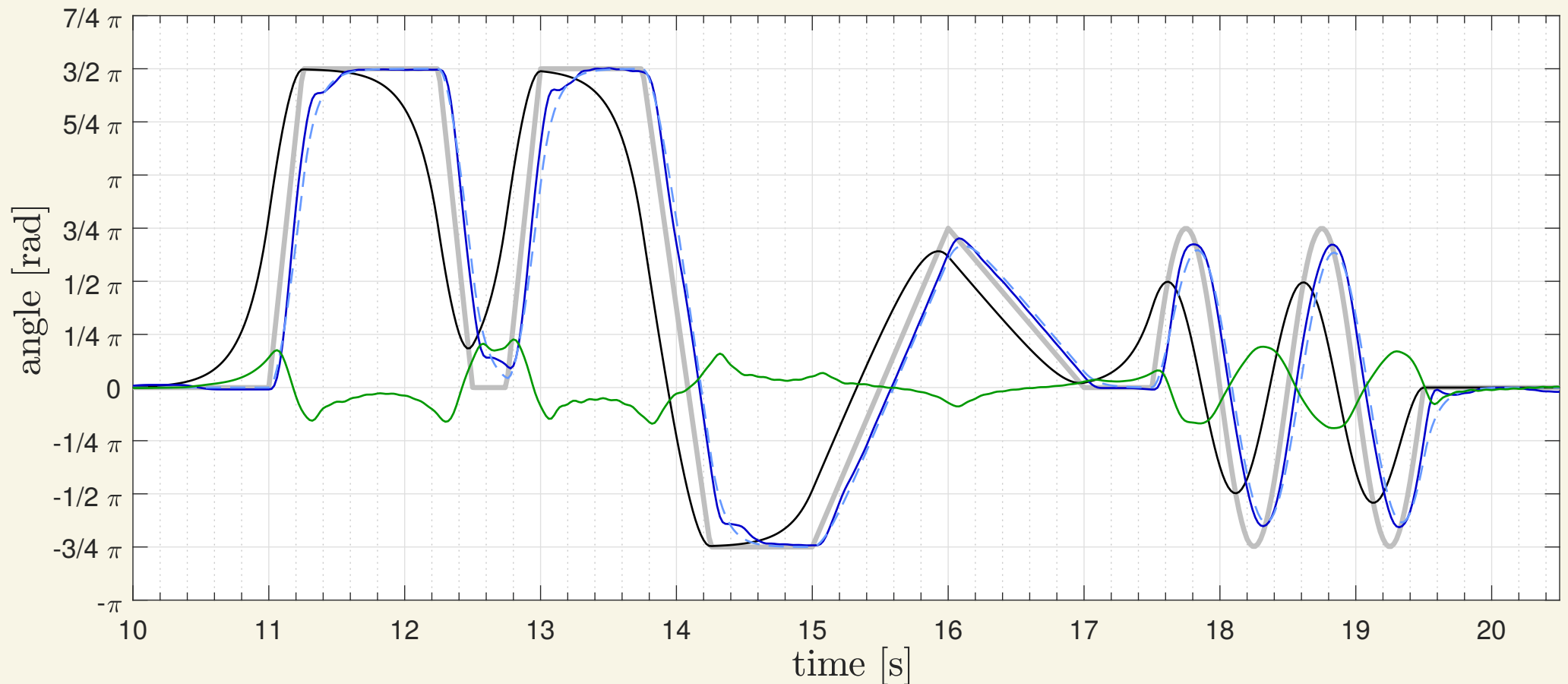
acausal filter

feedforward gains

$$u = \underline{q_{2c}} + \underline{\alpha_1} \underline{\dot{q}_{2c}} + \underline{\alpha_2} \underline{\ddot{q}_{2c}}$$

command signal and derivatives

# Experimental Results (Tippy balancing using crossbar)



- $q_c$
- $q_f$
- $q_2$
- - -  $q_t$
- $10q_1$

actual response  $q_2$  very closely follows  
theoretical response  $q_t$

Website: <http://royfeatherstone.org/skippy>

Acknowledgements: the video of Tippy and the experimental results were produced by members of the Skippy Team

THE END