Balancing on a Knife Edge While Doing Something Else

by Roy Featherstone

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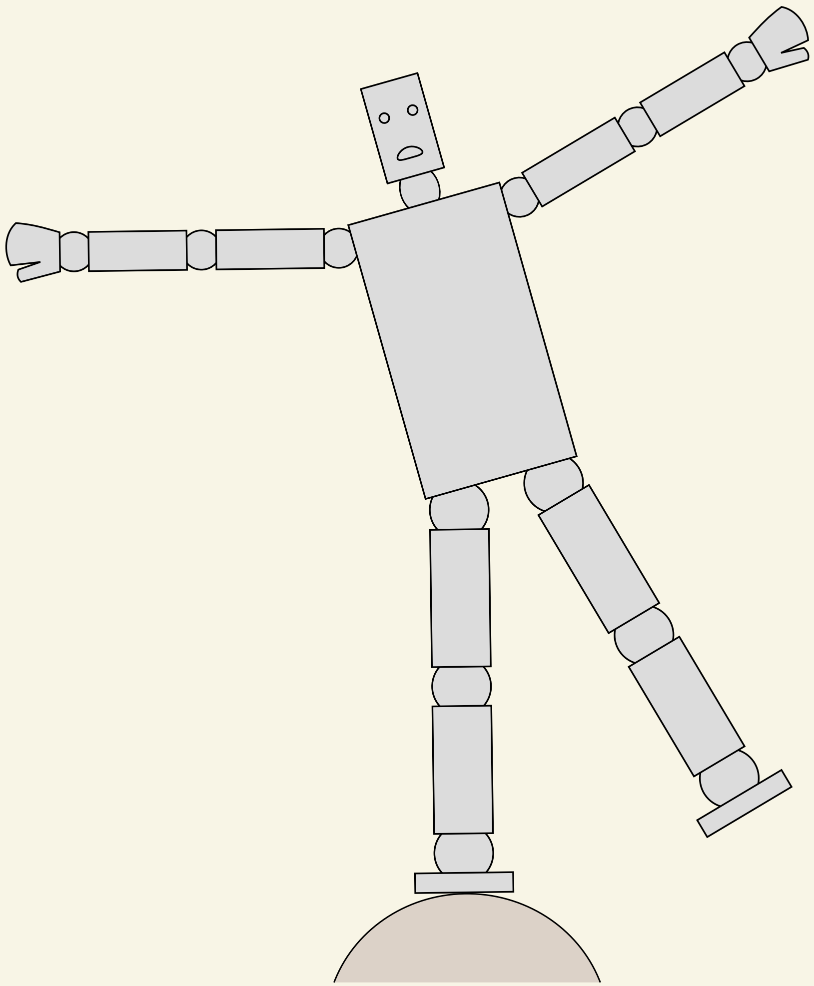
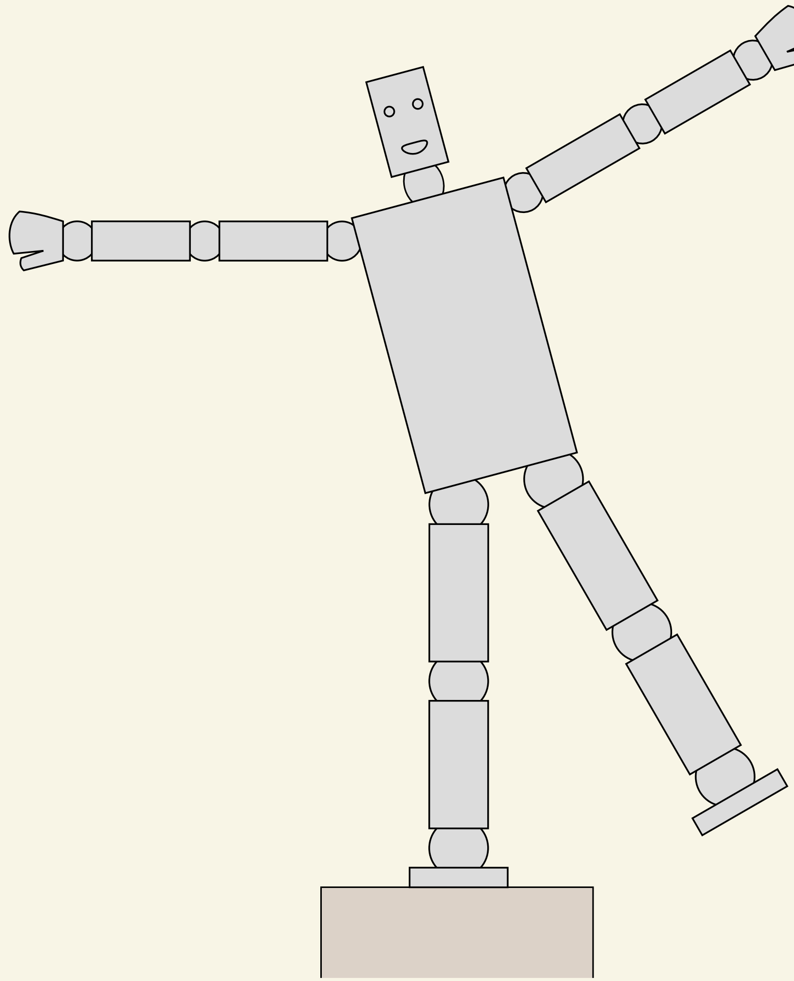
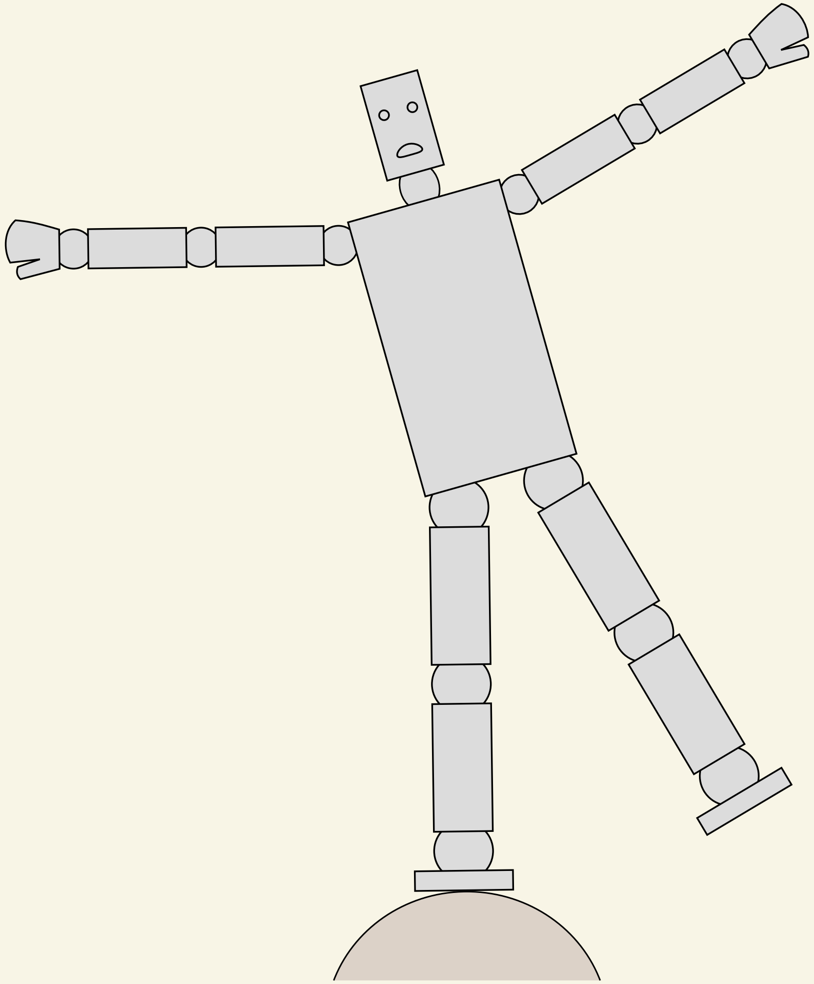


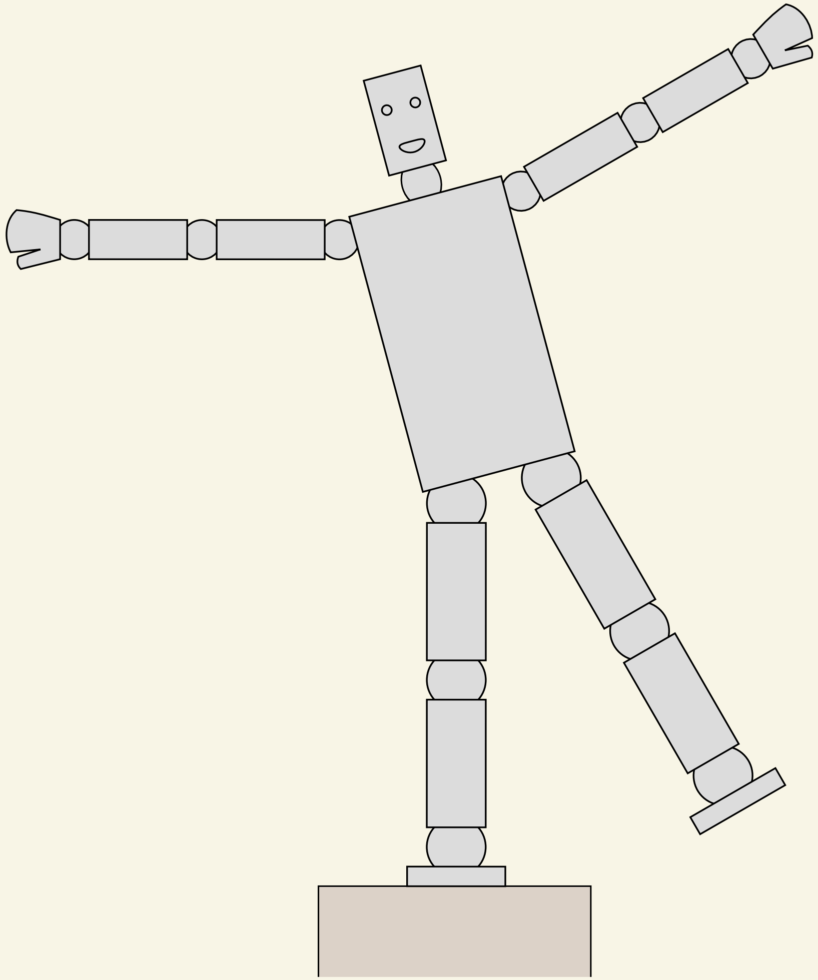
narrow support — effectively a single point or line



substantial polygon of support



dynamically unstable —
must balance actively



statically stable as long as
ground reaction force stays
inside support polygon

High-Performance Balancing

Definition:

A robot is a high-performance balancer if it can

- accurately follow commands to make large, fast movements without losing its balance; and
- quickly recover from large balance disturbances

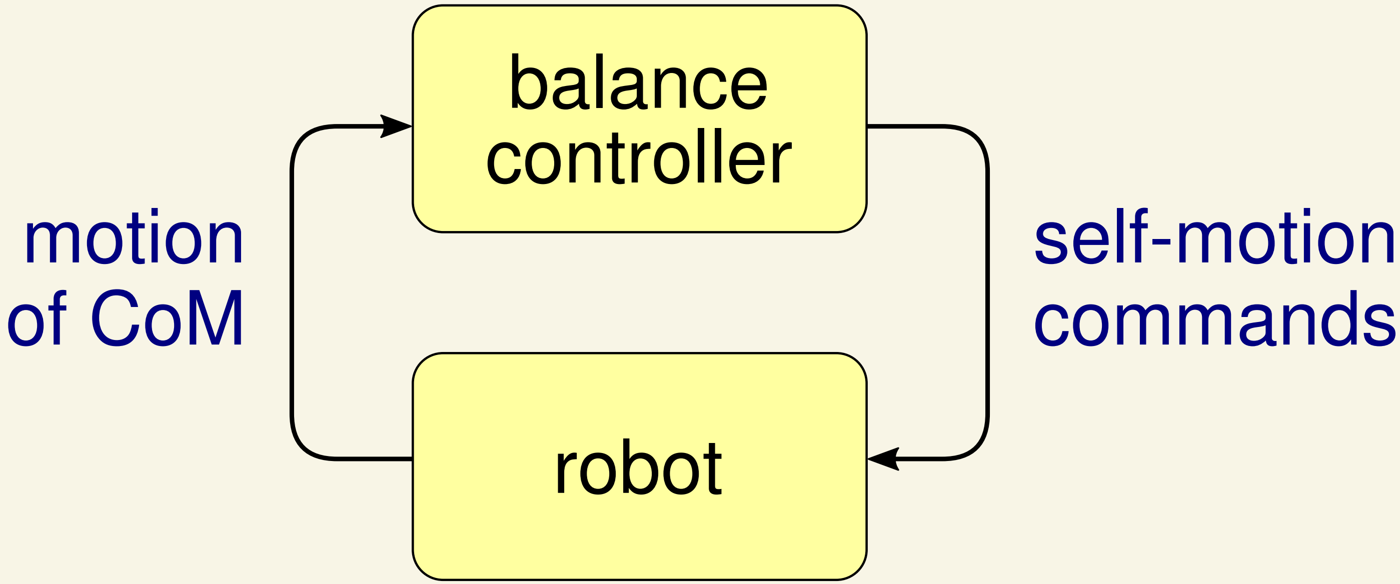
High-Performance Balancing

Our Approach:

1. Balancing is a physical activity, not just a control theory exercise;
   - so study the *physics of balancing*

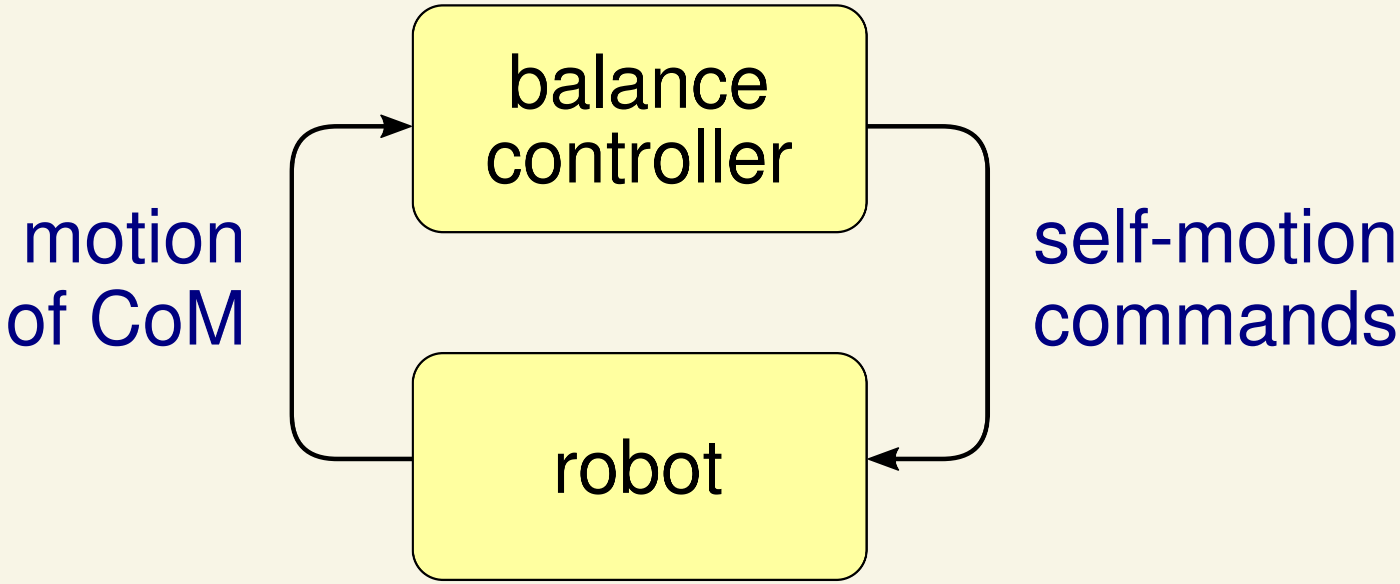
2. A robot's physical ability to balance is a property of the robot itself, not the control system;
   - so study what makes a robot good at balancing, and make your robot *good by design*.

Physical Ability to Balance



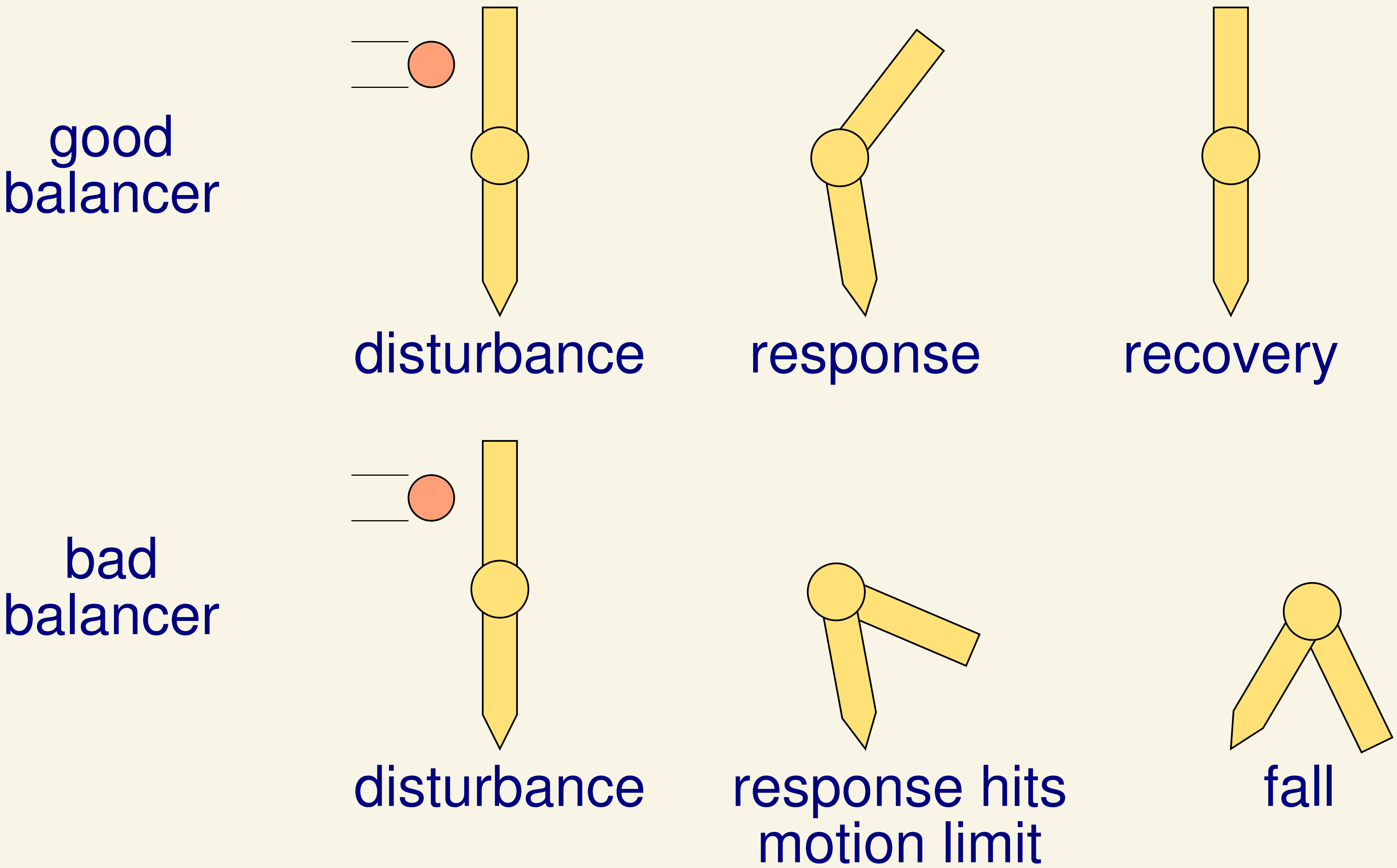
Balancing on a narrow support requires moving the robot's centre of mass (CoM) relative to the support by means of self motions (i.e., motions of the actuated joints).

Physical Ability to Balance



So a robot is *physically good at balancing* if a relatively small self motion causes a relatively large CoM motion.

Physical Ability to Balance



Physical Ability to Balance

This leads to the idea of ***velocity gain***:

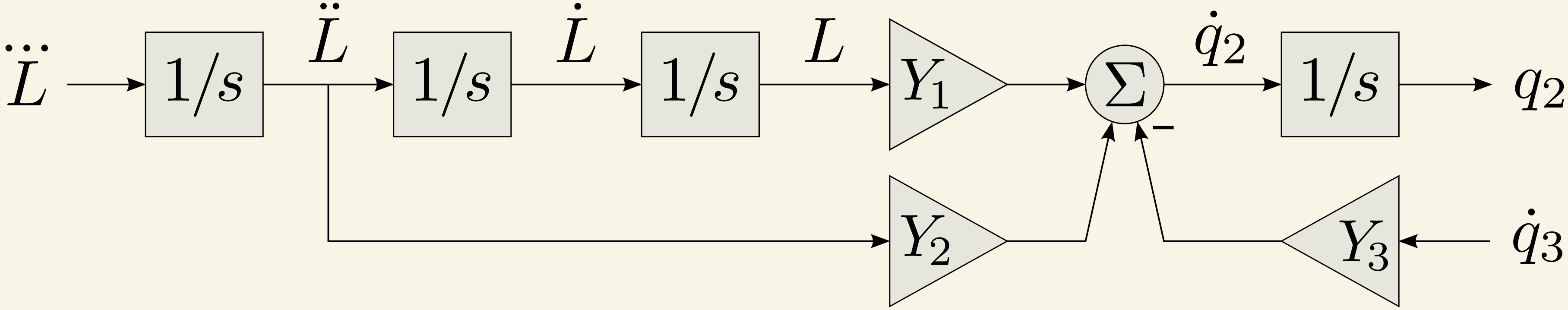
$$\frac{\Delta \boldsymbol{v}_{\mathrm{CoM}}}{\Delta \dot{q}_{\mathrm{a}}}$$

step change in CoM velocity

step change in actuated joint velocity

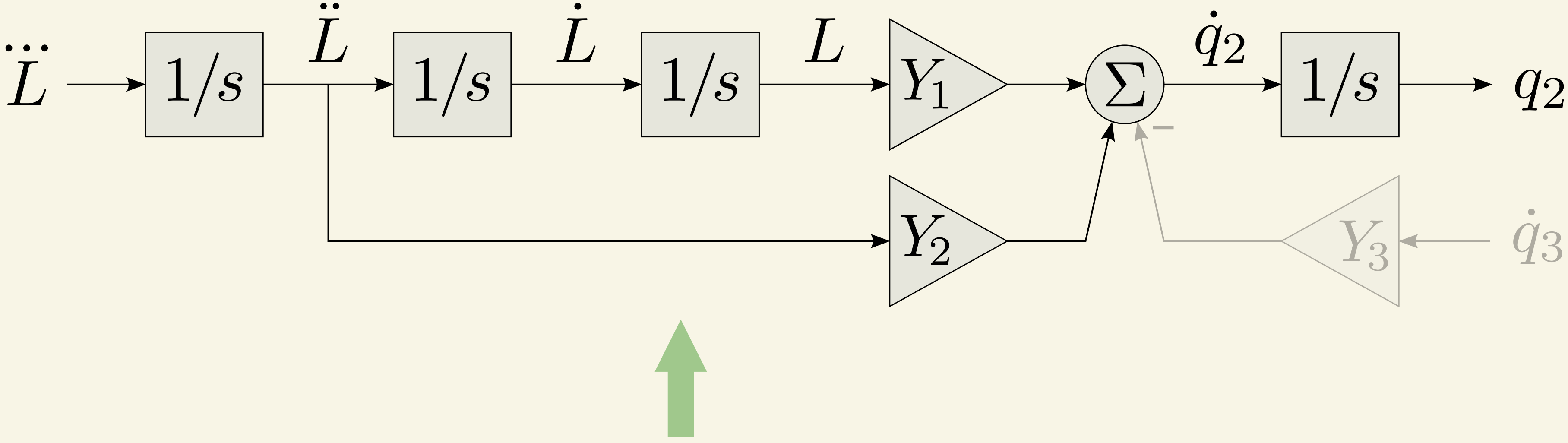
both caused by an impulse at the actuated joint

The Simple Dynamics of Balancing



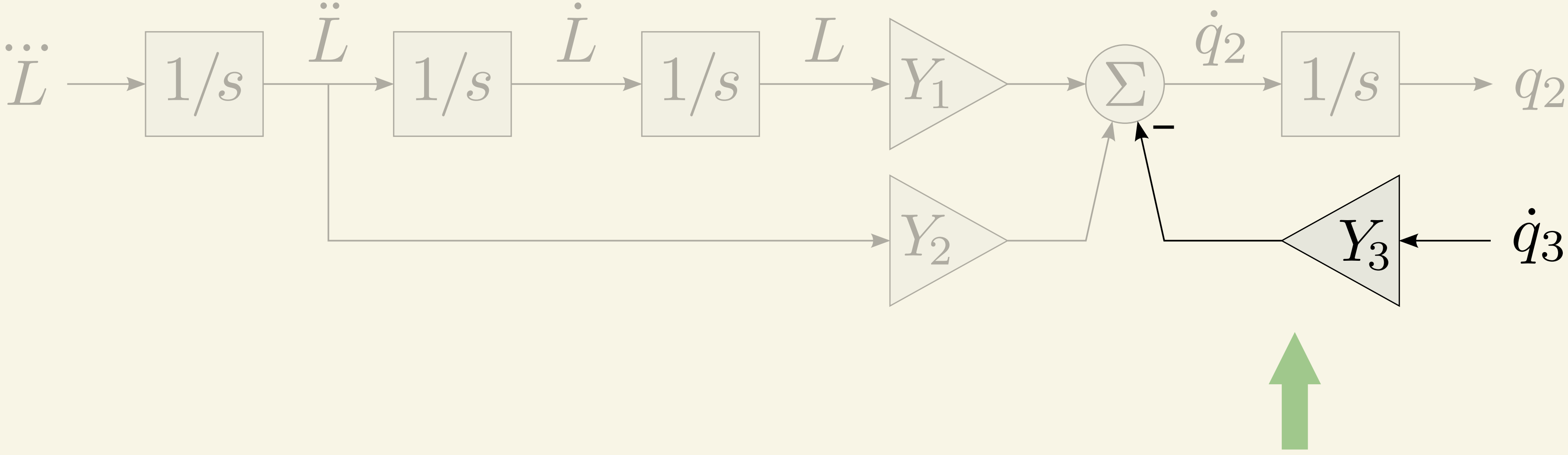
The physical process of balancing can be expressed in the form of this block diagram, which serves as the *plant* for the balance controller to control.

The Simple Dynamics of Balancing



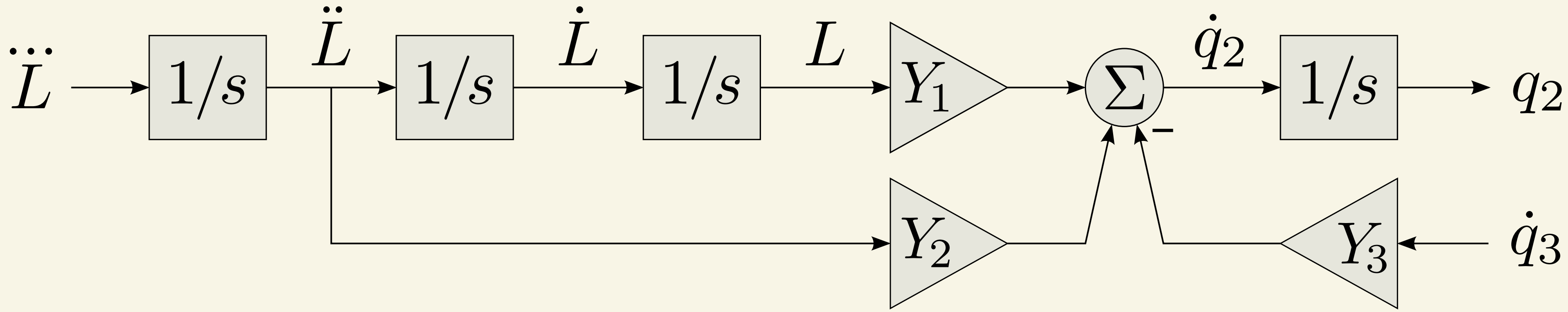
This portion is concerned with balancing

The Simple Dynamics of Balancing



and this portion describes the disturbances caused by other motions.

The Simple Dynamics of Balancing



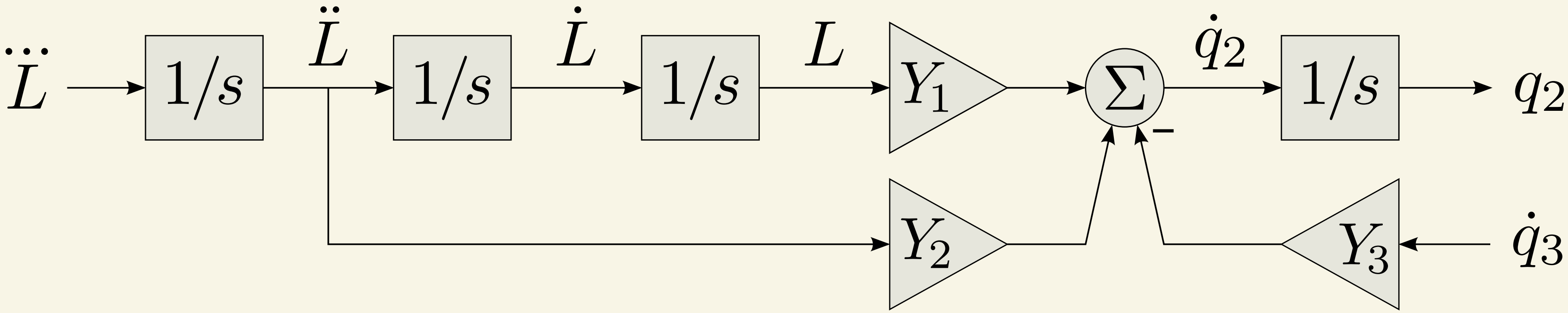
L angular momentum of the robot about the support

q_1 variable describing the robot's angle relative to the ground (passive revolute joint)

q_2 *overloaded* variable used *both* to balance *and* to follow motion commands

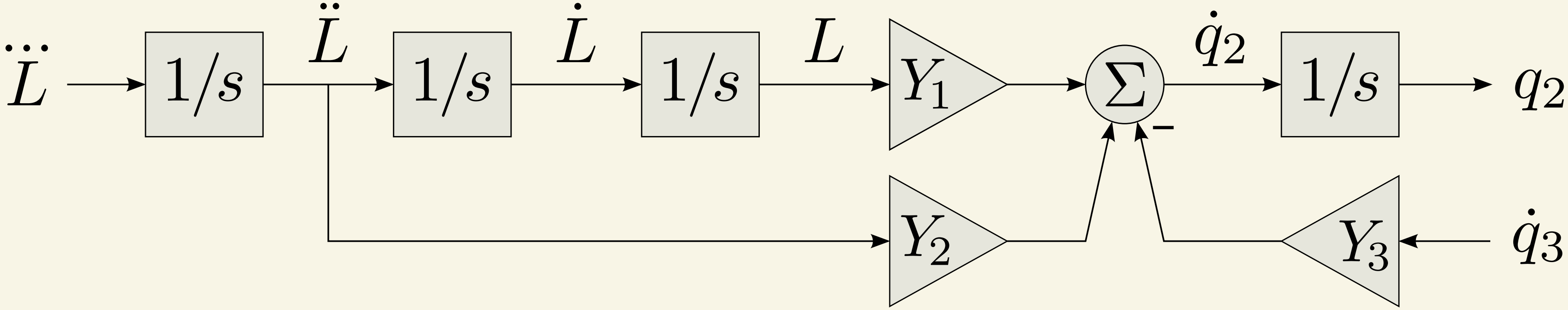
q_3 all other position variables

The Simple Dynamics of Balancing



The balancing behaviour of the robot depends only on the two gains Y_1 and Y_2 which are easily calculated functions of the robot's configuration variables.

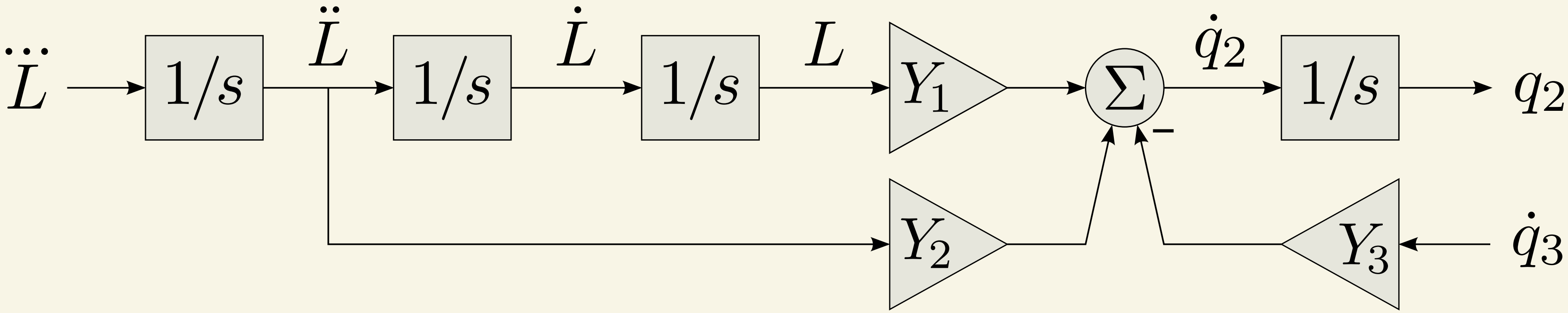
The Simple Dynamics of Balancing



Y_1 and Y_2 are functions of two easily measured physical properties of the robot:

- the natural time constant of toppling, T_c, and
- the linear velocity gain of $\dot{q}_2$

The Simple Dynamics of Balancing



the rate at which the robot begins to fall if q_2 is held constant

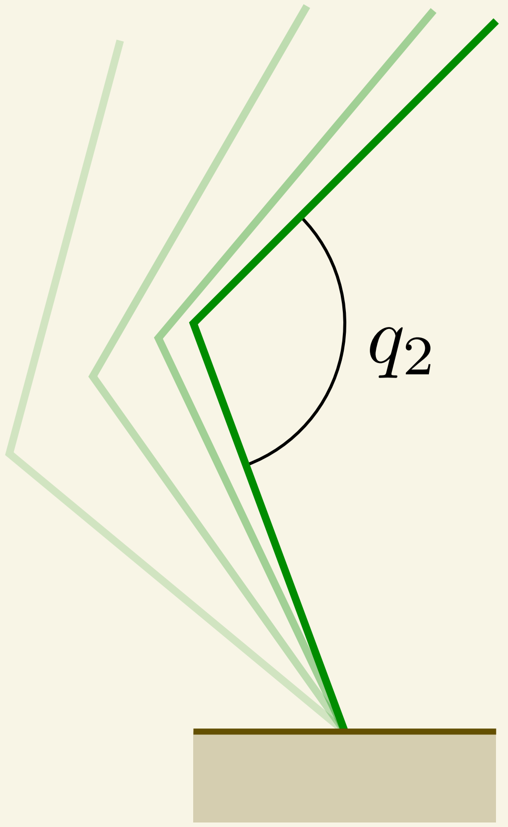
Y_1 and wo easily measured physical propert

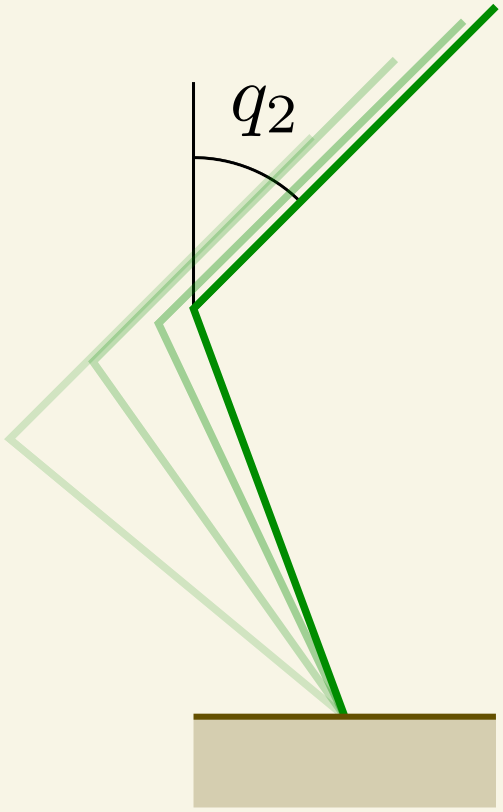
- the natural time constant of toppling, T and
- the linear velocity gain of $\dot{q}_2$

the step change in CoM velocity caused by a unit step change in $\dot{q}_2$

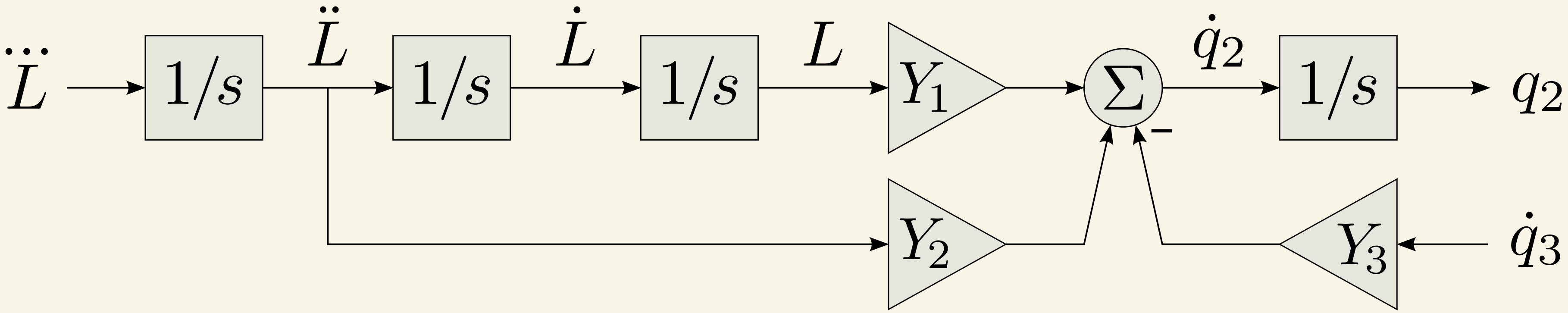
Different Ways to Fall

The time constant of toppling, T_c, is the rate at which the robot would start to fall if q_2 is held constant. It therefore depends on the definition of q_2. Here are two examples:





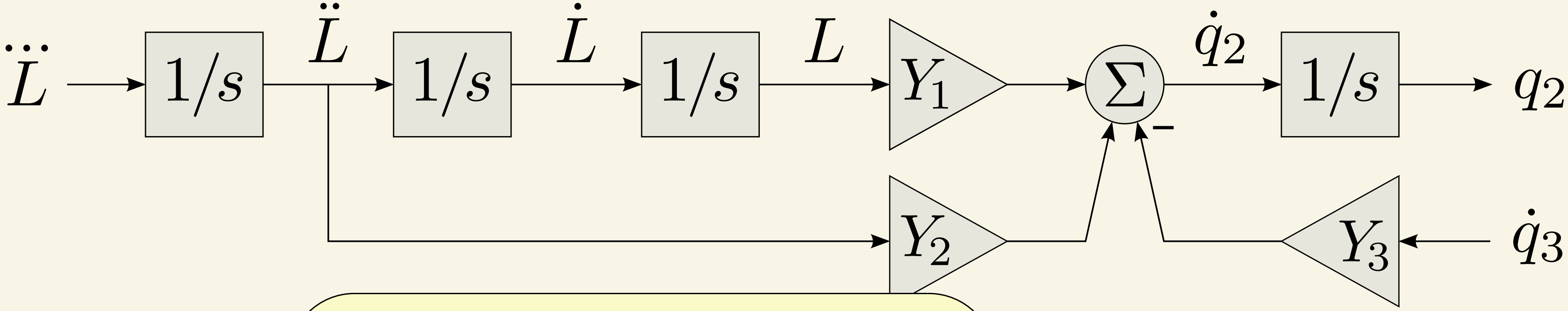
Simple Balance Control



High-performance balancing can be achieved using

- a simple control law closed around *this plant*
- and a simple *acausal filter* that implements *leaning in anticipation* of the balance disturbances that will be caused by commanded future motions.

Simple Balance Control



a first-order low-pass filter running *backwards* in time from the future to the present

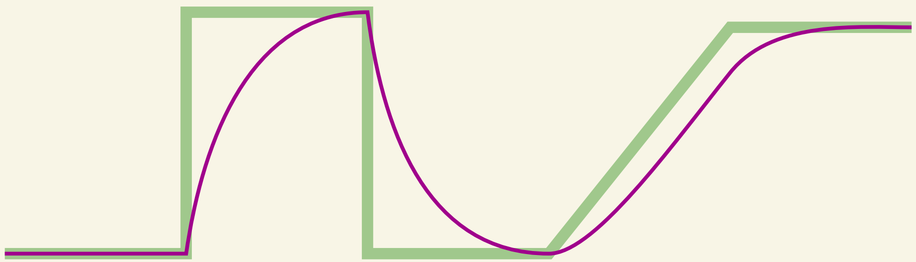
High-perfor… achieved using

- a simple … nd *this plant*
- and a simple *acausal filter* that implements *leaning in anticipation* of the balance disturbances that will be caused by commanded future motions.

The Acausal Filter

a first-order low-pass filter running backwards in time
to cancel the non-minimum-phase response of the robot

normal low-pass filter

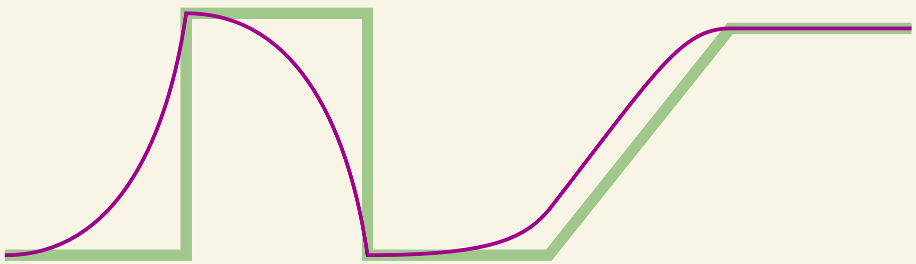


transfer function

$$\frac{1}{1+T_c s}$$

natural time constant of toppling

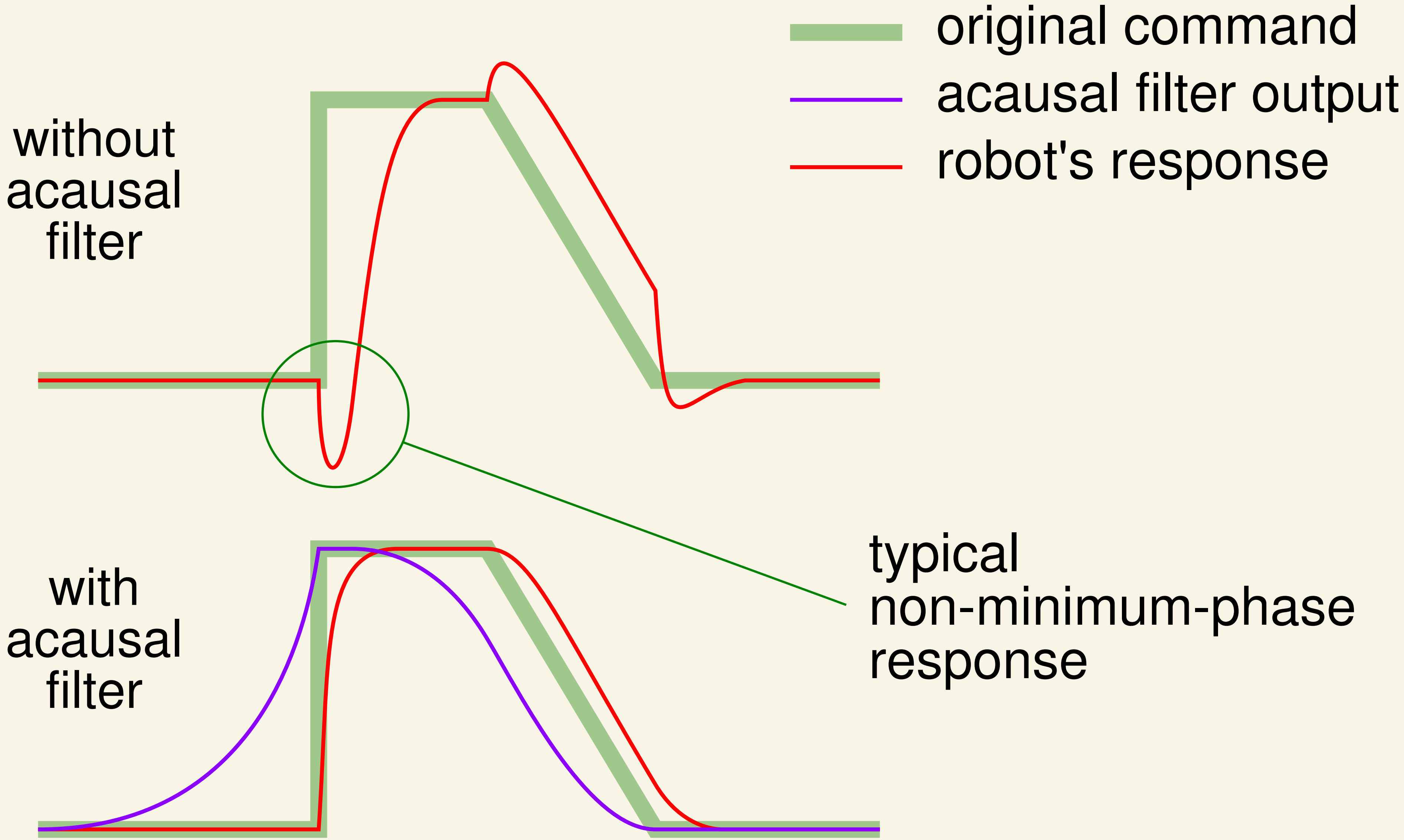
acausal low-pass filter



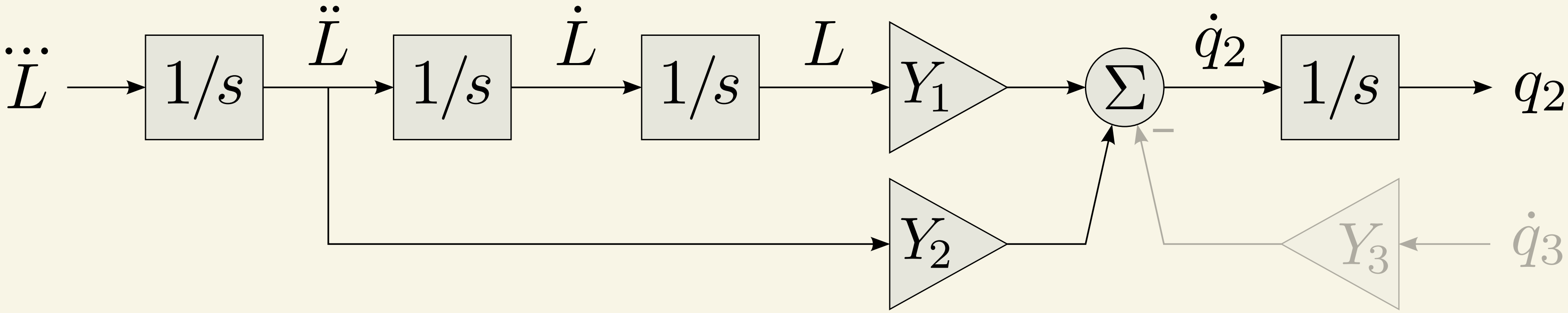
$$\frac{1}{1-T_c s}$$

unstable pole in forward time

The Acausal Filter

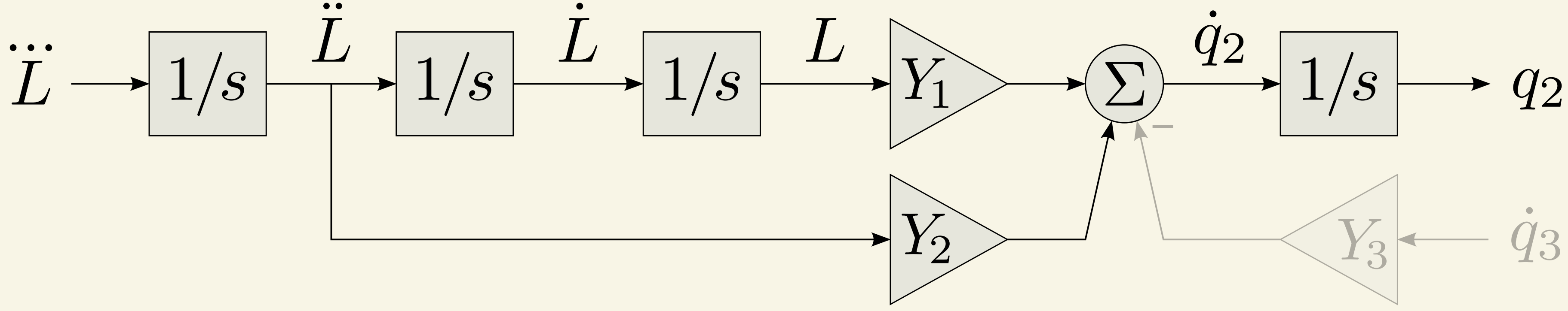


Balance Control Law



$$\dddot{L} = k_{dd}\ddot{L} + k_d\dot{L} + k_L L + k_q(q_2 - AF(u))$$

Balance Control Law



$$\dddot{L} = \underline{k_{dd}}\ddot{L} + \underline{k_d}\dot{L} + \underline{k_L}L + \underline{k_q}(q_2 - AF(u))$$

feedback gains

acausal filter

feedforward gains

$$u = \underline{q_{2c}} + \alpha_1 \underline{\dot{q}_{2c}} + \alpha_2 \underline{\ddot{q}_{2c}}$$

command signal and derivatives