A New Simple Model of Balancing in the Plane

Roy Featherstone

ISTITUTO ITALIANO DI TECNOLOGIA
ADVANCED ROBOTICS

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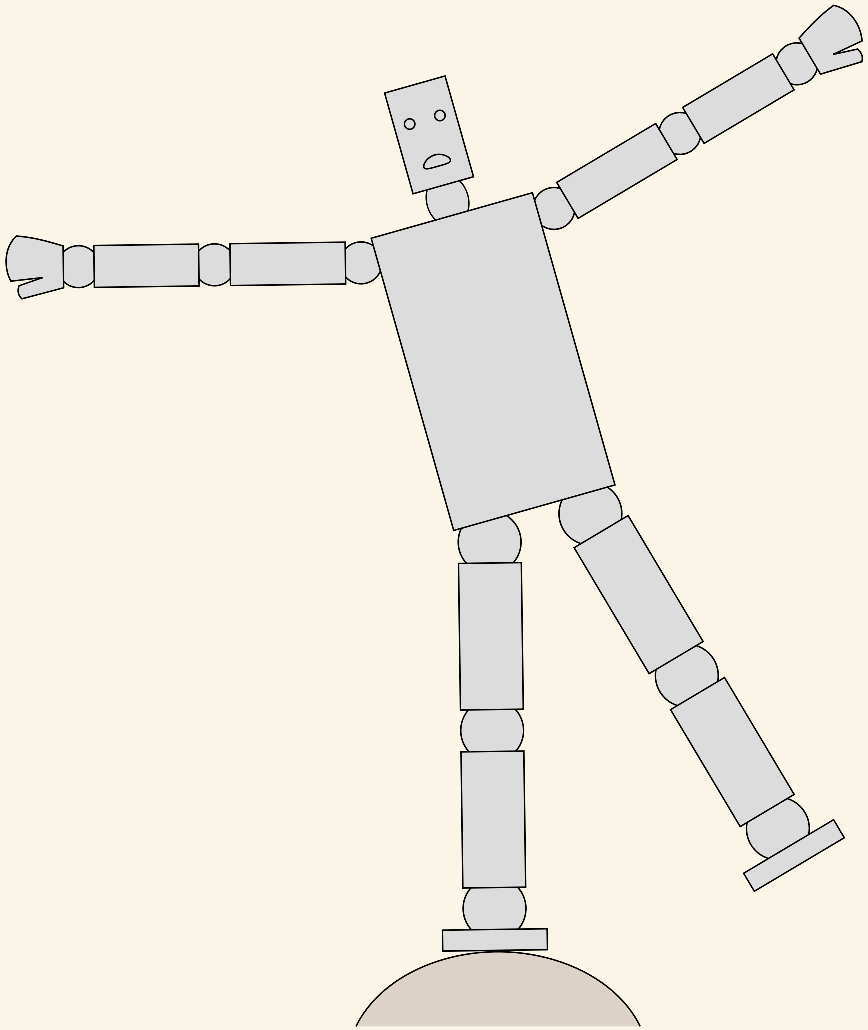
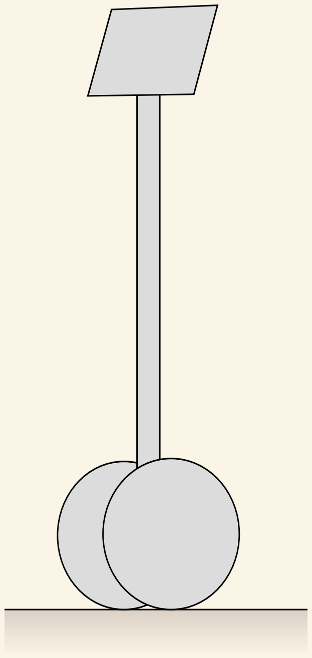
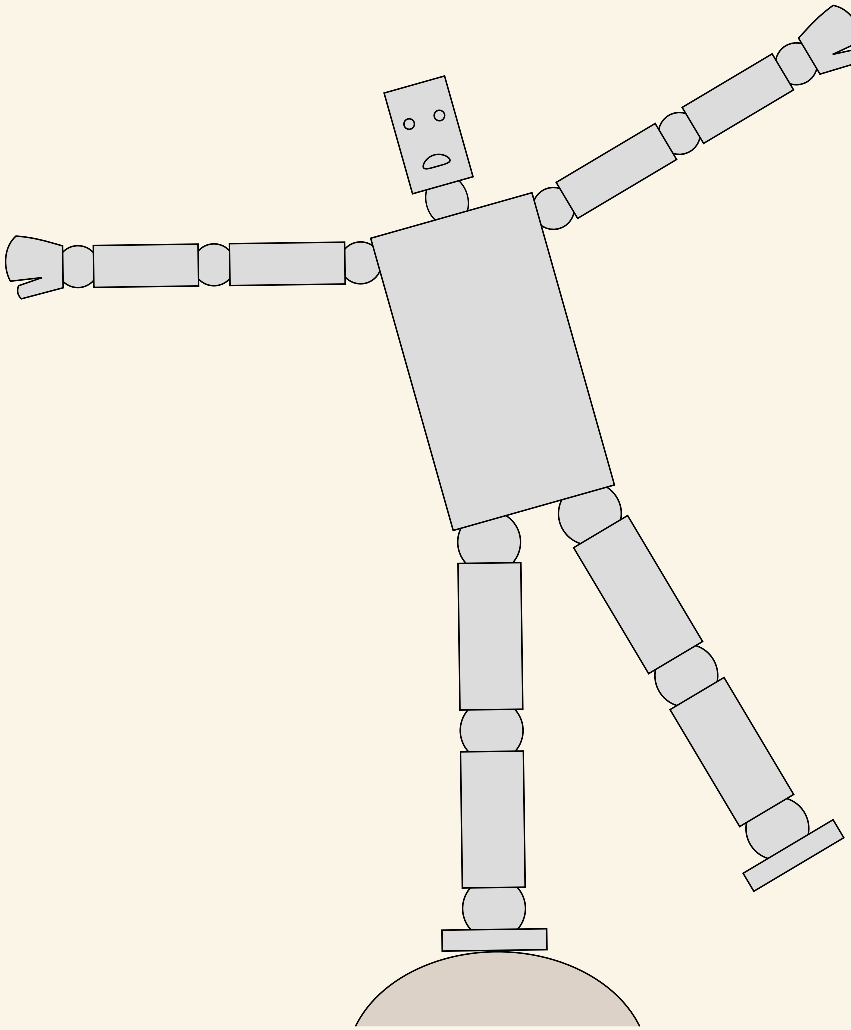


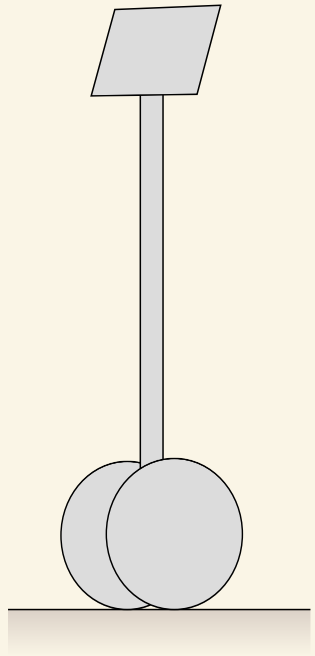
Robots do not always have a polygon of support.

Sometimes they have to balance actively.

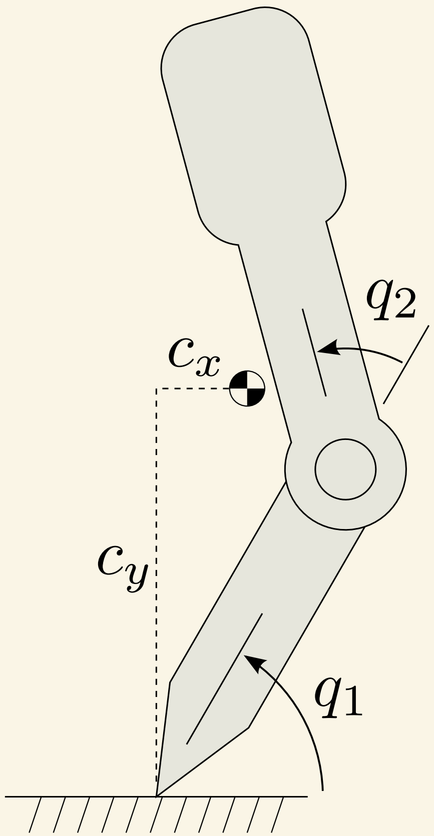




Balancing is usually seen as a control problem, but it is also a *physical process*, and can be analysed as such.



Physics of Balancing on a Point



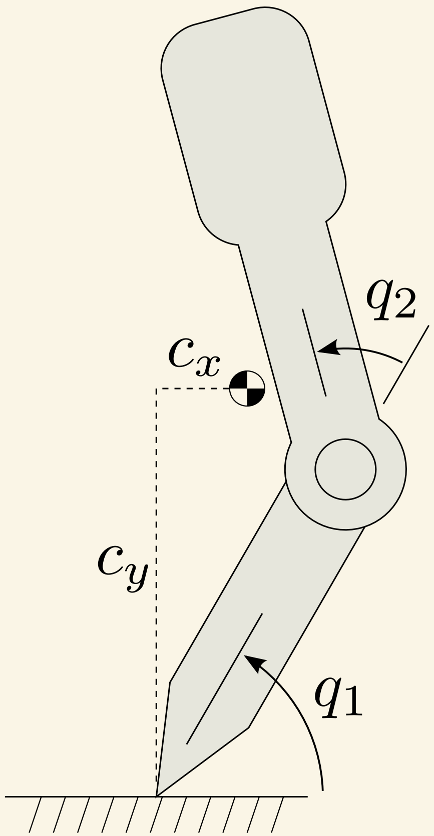
Objectives:

1. Maintain balance: $c_x = \dot{c}_x = 0$
2. Follow commanded motion: $q_2 = q_{2c}$, $\dot{q}_2 = \dot{q}_{2c}$

The control problem:

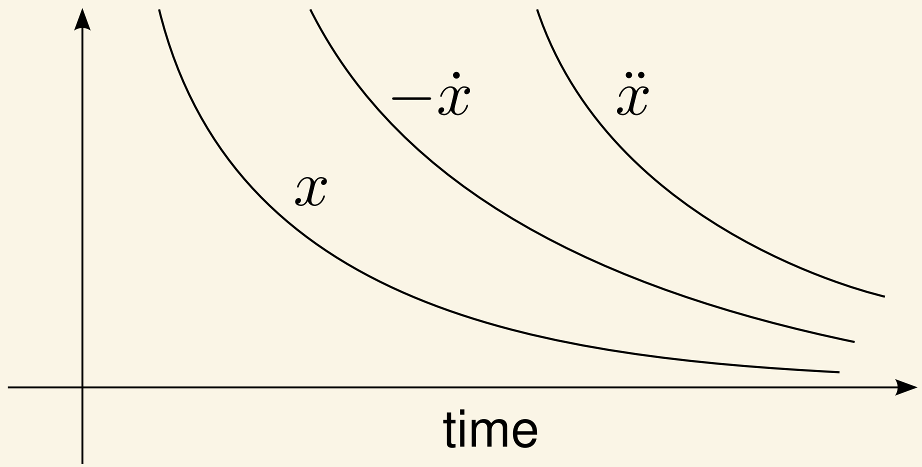
The controller must control 4 variables (c_x, $\dot{c}_x$, q_2 and $\dot{q}_2$), but has direct control of only one variable: τ_2

Physics of Balancing on a Point



The control solution: (in principle)

If a control system succeeds in driving a variable x to zero, then a side-effect is to drive $\dot{x}$, $\ddot{x}$, etc. also to zero.



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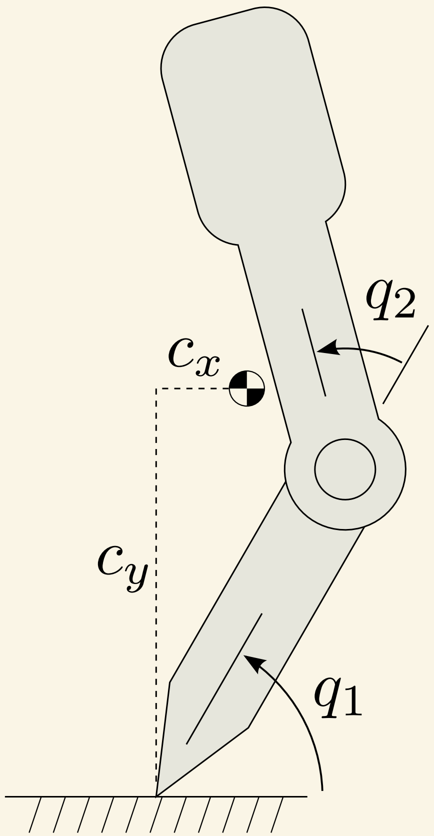
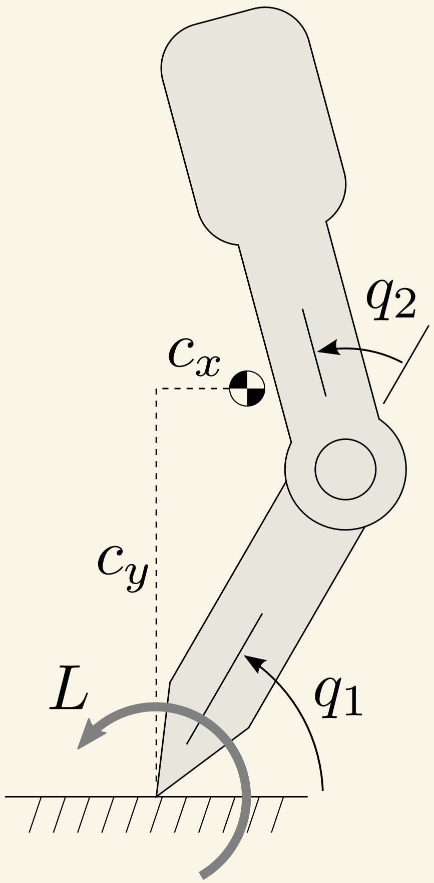


The control solution: (in principle)

So we seek a new set of state variables to use in place of q_1, q_2, $\dot{q}_1$ and $\dot{q}_2$ with the property that controlling one has the side-effect of controlling the other three.

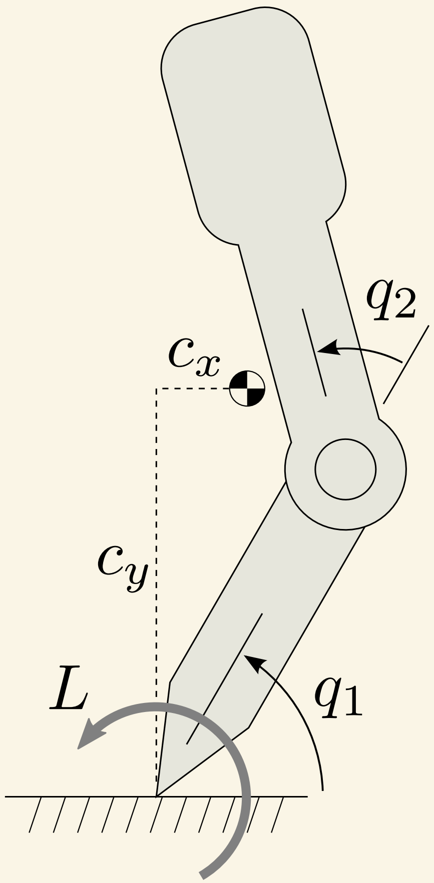
Physics of Balancing on a Point



Analysis:

Let L be the angular momentum of the robot about the support point. L has the special property that $\dot{L}$ is the moment of gravity about the support point.

Physics of Balancing on a Point



Analysis:

$$L = H_{11}\dot{q}_1 + H_{12}\dot{q}_2$$

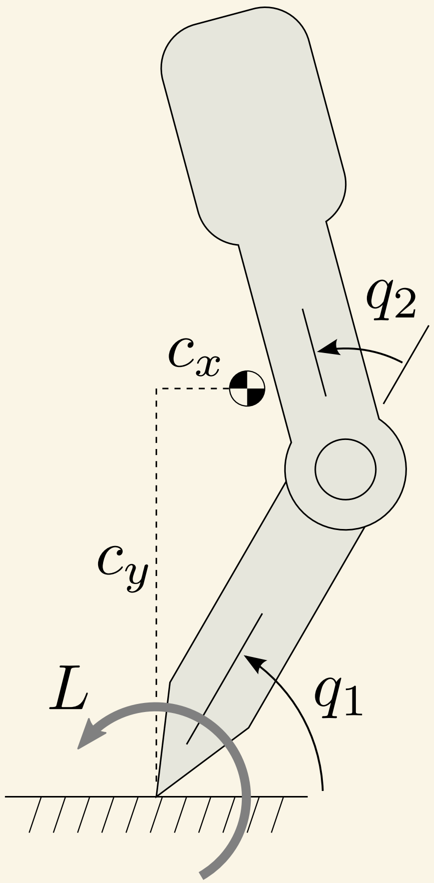
$$\dot{L} = -mgc_x$$

$$\ddot{L} = -mg\dot{c}_x$$

Where H_{ij} are elements of the joint-space inertia matrix, m is the mass of the robot, and g is the acceleration of gravity.

Observe that L and $\ddot{L}$ are linear functions of velocity. . . .

Physics of Balancing on a Point



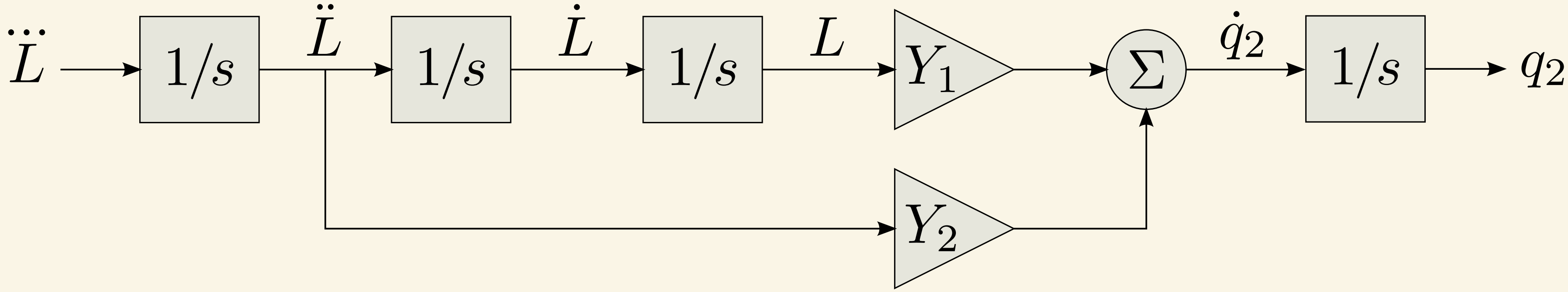
Analysis:

. . . . so it is possible to express $\dot{q}_2$ as a linear function of L and $\ddot{L}$:

$$\dot{q}_2 = Y_1 L + Y_2 \ddot{L}$$

where Y_1 and Y_2 are functions of q_1 and q_2 only, and can be calculated easily via standard dynamics algorithms.

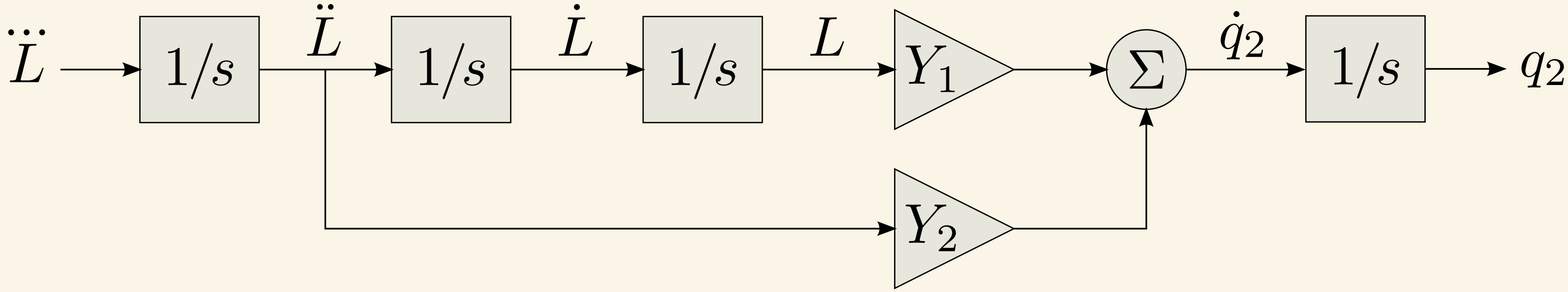
New Model of Balancing



The result is a new model of the balancing behaviour of the robot in which

- the state variables are $\ddot{L}$, $\dot{L}$, L and q_2,
- the input is $\dddot{L}$ and the output is q_2,
- controlling q_2 has the side-effect of maintaining the robot's balance

New Model of Balancing



$$\begin{aligned} q_2 = \text{const} &\Rightarrow \dot{q}_2 = 0 \\ \dot{q}_2 = 0 &\Rightarrow L = \dot{L} = \ddot{L} = 0 \\ \dot{L} = 0 &\Rightarrow c_x = 0 \\ \ddot{L} = 0 &\Rightarrow \dot{c}_x = 0 \end{aligned}$$

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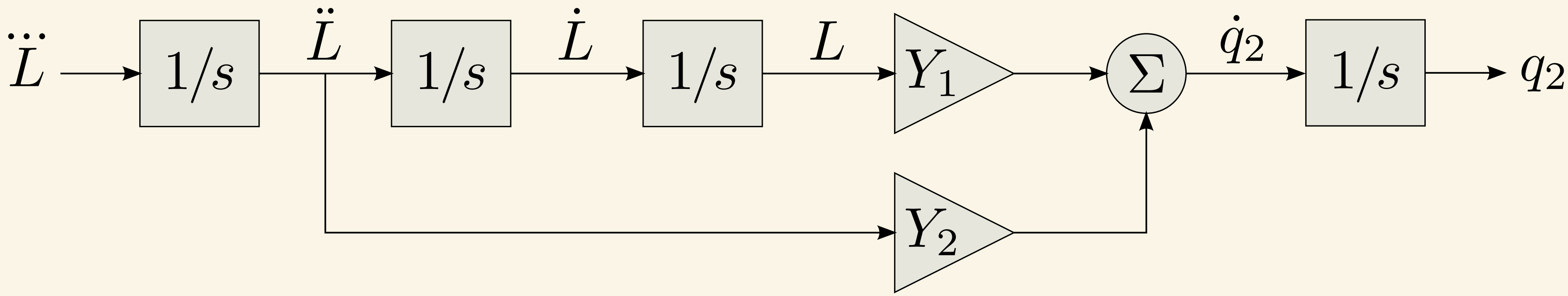
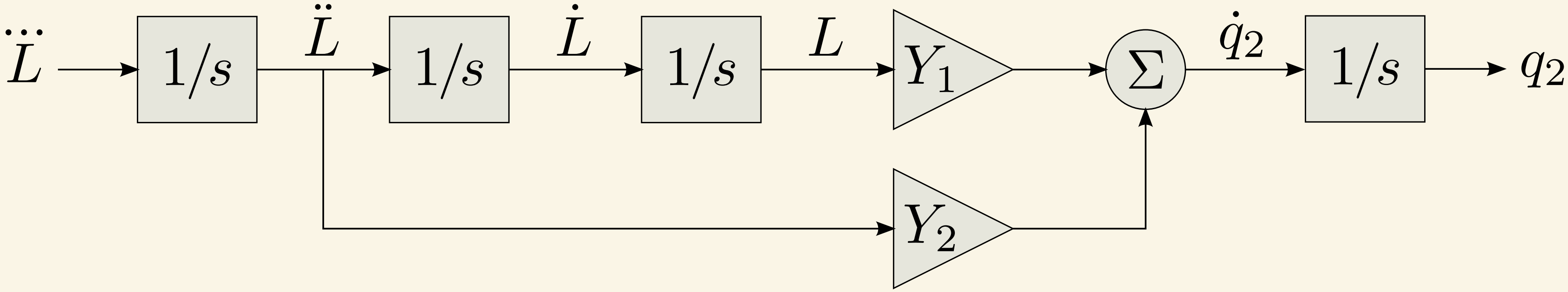
New Model of Balancing



To control the robot we

1. map q_1, $\dot{q}_1$, q_2 and $\dot{q}_2$ to $\ddot{L}$, $\dot{L}$, L and q_2,
2. apply a simple control law to calculate $\dddot{L}$,
3. convert $\dddot{L}$ to τ_2 or $\ddot{q}_2$ as required

Balance Controller



balance control law

$$\dddot{L} = k_{dd}(\ddot{L} - \ddot{L}_{\mathrm{c}}) + k_d(\dot{L} - \dot{L}_{\mathrm{c}}) + k_L(L - L_{\mathrm{c}}) + k_q(q_2 - q_{2\mathrm{c}})$$

optional

The gains depend only on Y_1, Y_2 and the user's choice of poles.

More Details

- the basic idea extends to the case of a general planar robot
- it also extends to 3D (work in progress)
- the computational cost is small, and only standard dynamics calculations are needed
- in the general case, any desired movement of the robot can be used for balancing, and all other movements can be devoted to other tasks