Quantitative Measures of a Robot's Ability to Balance

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ADVANCED ROBOTICS

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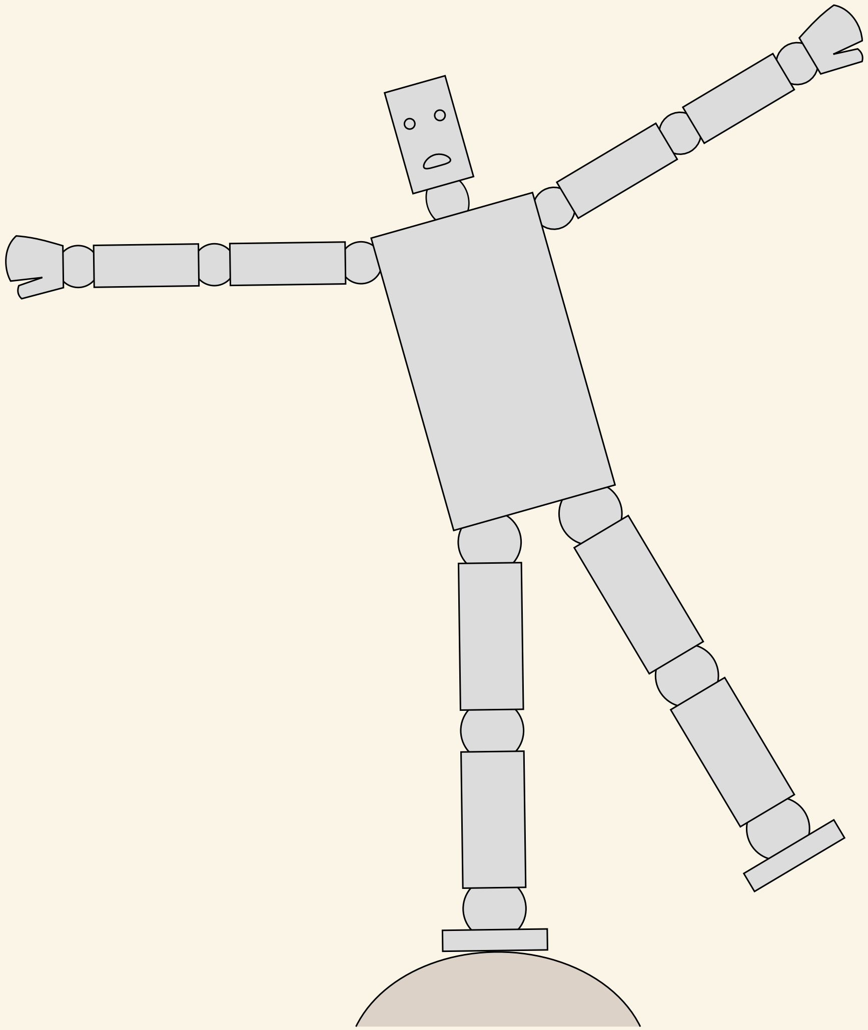
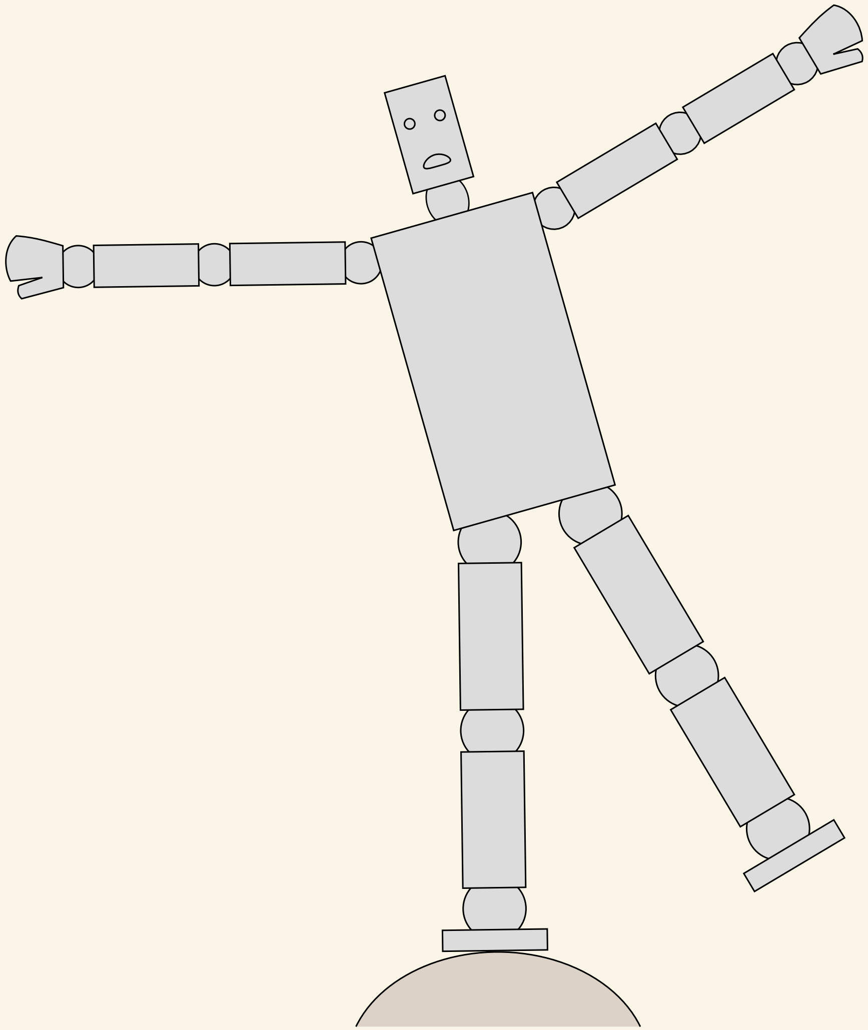


Problem:

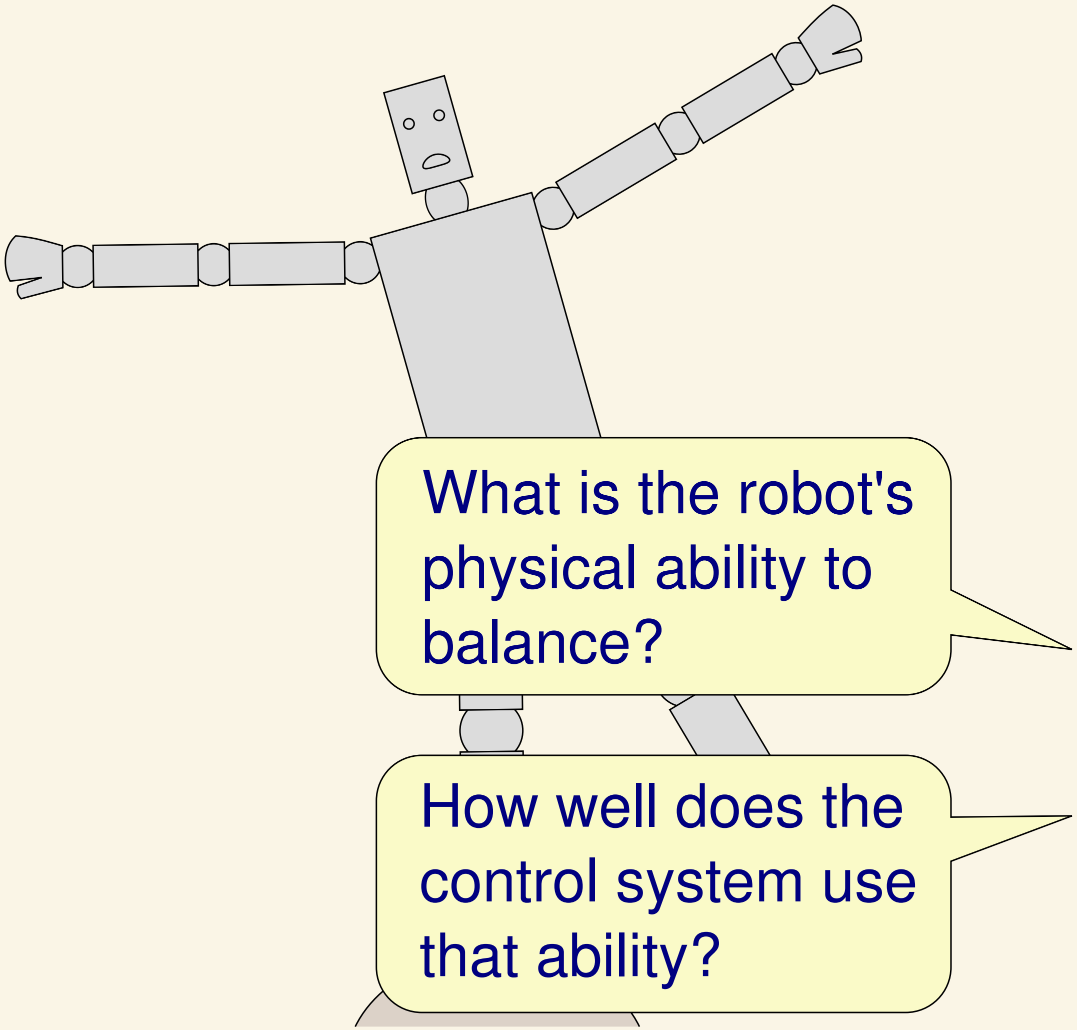
Will the robot fall over if

- the IMU noise is 0.5°?
- the IMU noise is 1°?
- the IMU noise is 2°?
- the ground moves by 2cm?



To answer questions like this, we need a *measure* of how good the robot is at balancing. There are two aspects:

- how good is the robot?
- how good is the control system?



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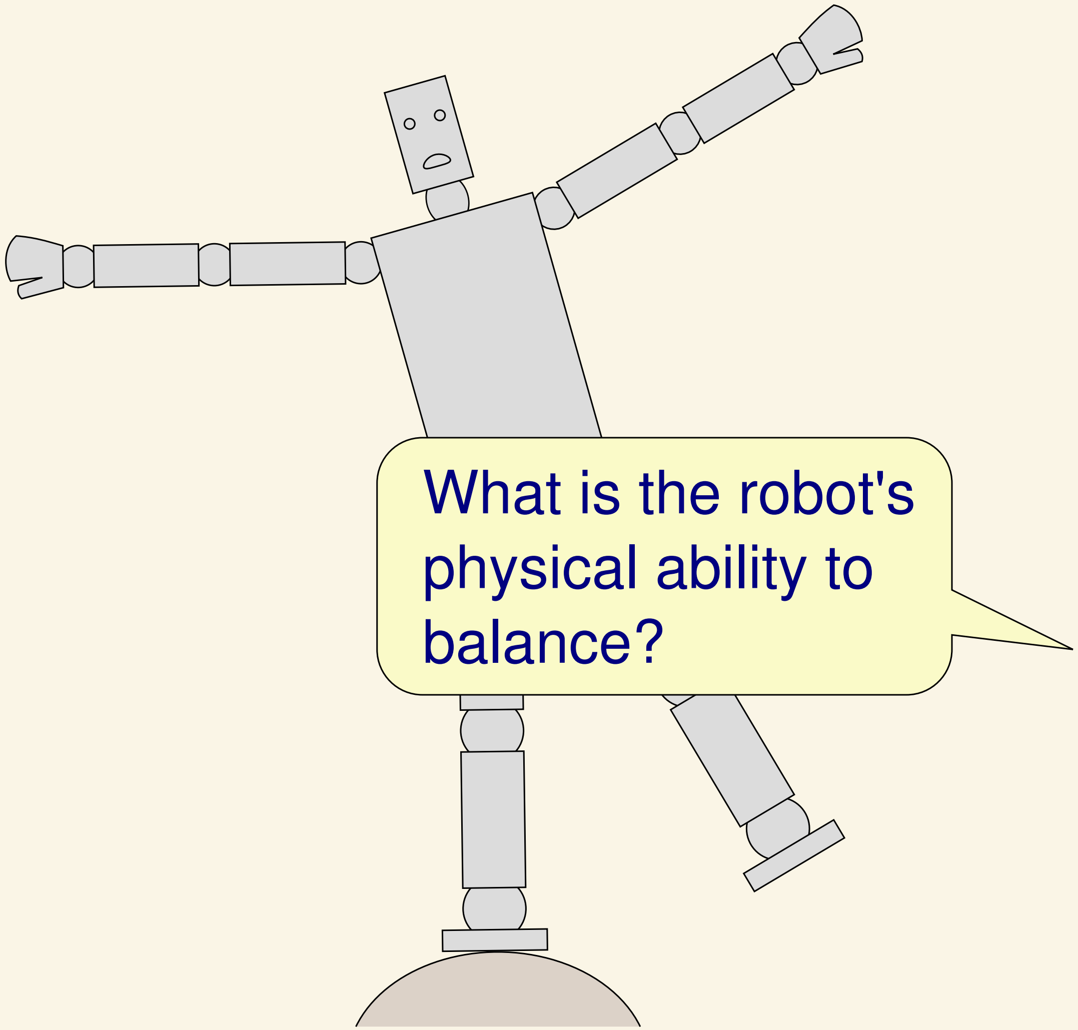

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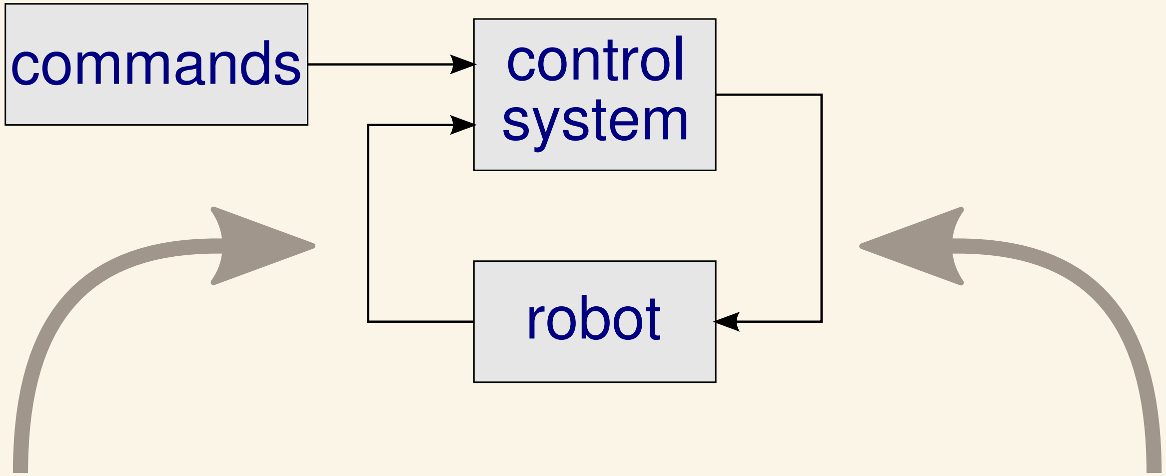
How do you define a robot's physical ability to balance?



The thing that the control system is trying to control is the motion of the robot's centre of mass (CoM).

The thing that the control system can control directly is the motion of the actuated joints.

How do you define a robot's physical ability to balance?

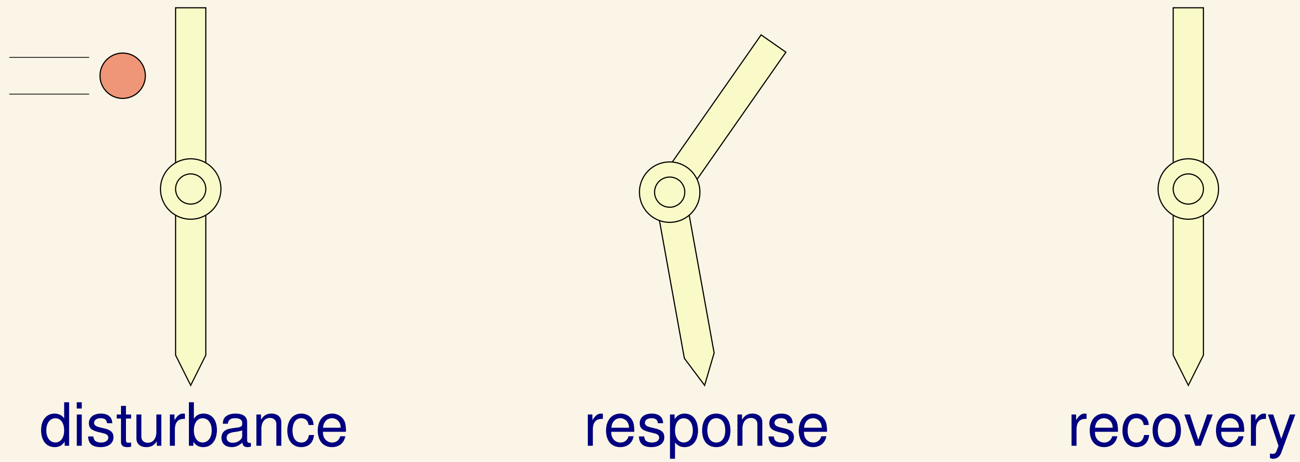
motion of actuated joints ⇨ motion of CoM

A robot is *good* at balancing if motion of the actuated joints has a *large* influence on the motion of the CoM; and the *magnitude* of this influence provides a *measure* of the robot's physical ability to balance.

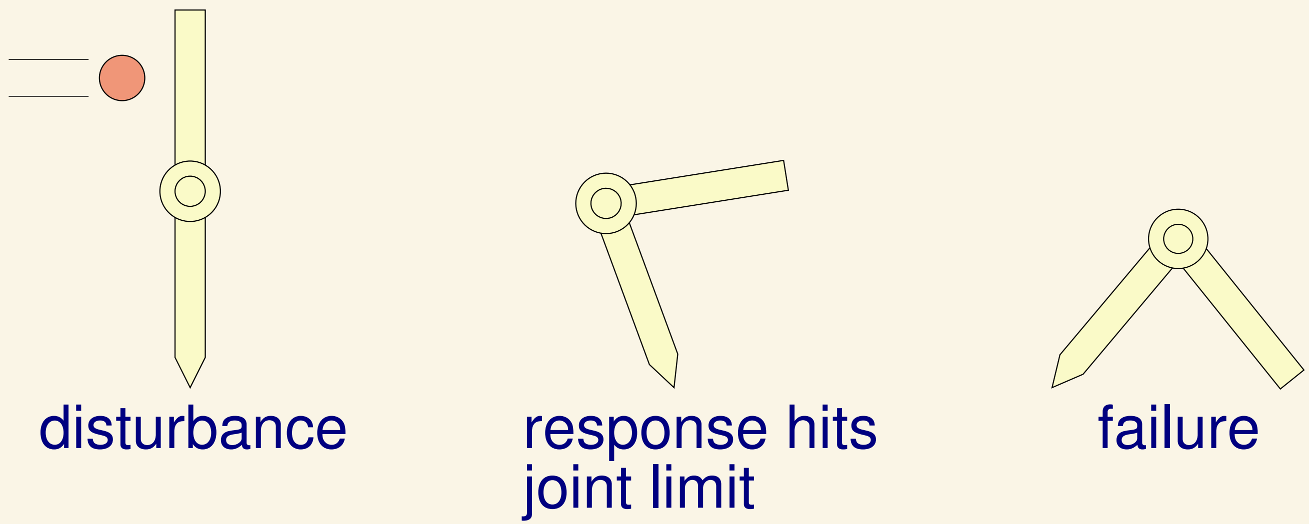
Thus, a robot that is *good* at balancing makes a *small* movement to correct a balance error, but a robot that is *bad* at balancing must make a *bigger* movement to correct the same error.

This is a *physical property of the mechanism*, and is therefore independent of the choice of control law.

Good Balancer

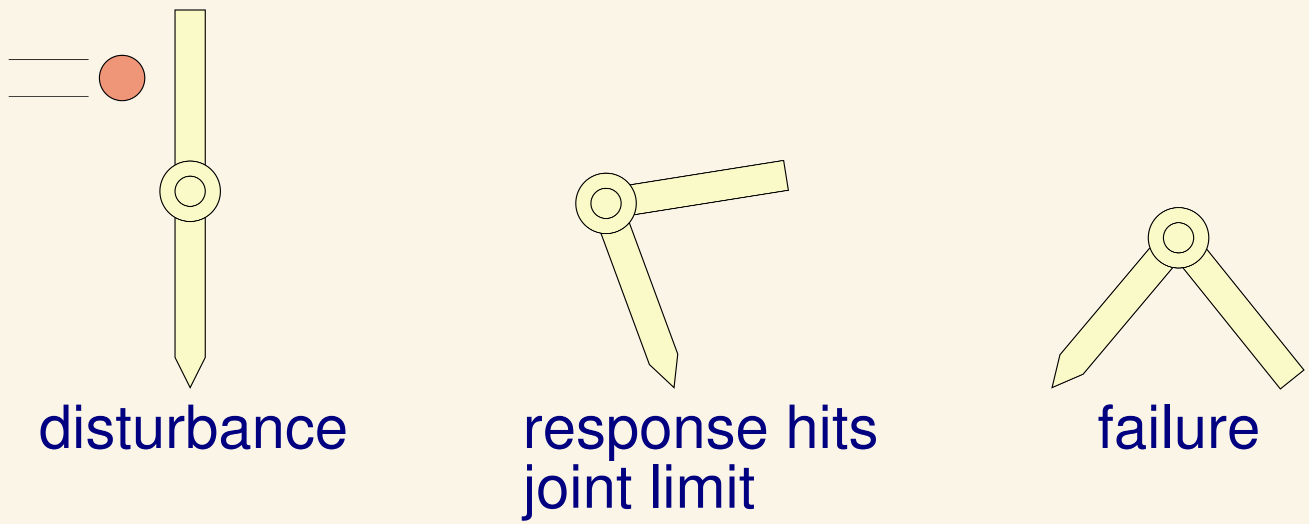


Bad Balancer

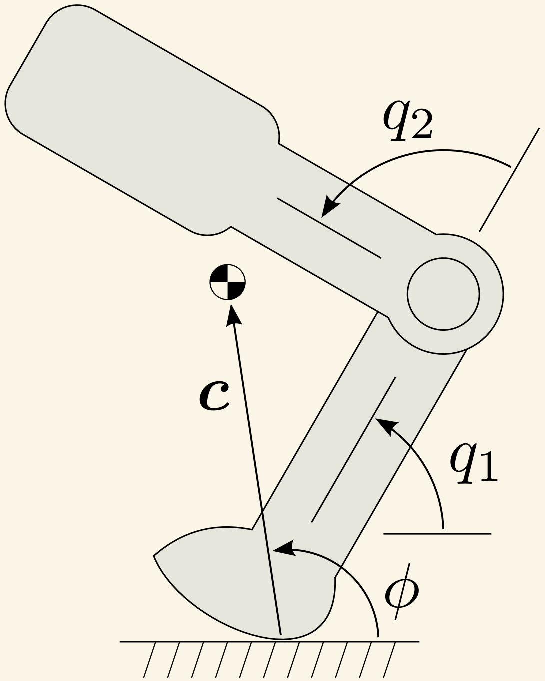


No control system can fix this problem:
it is a performance limit of the mechanism,
not the control system.

Bad
Balancer



Velocity Gain



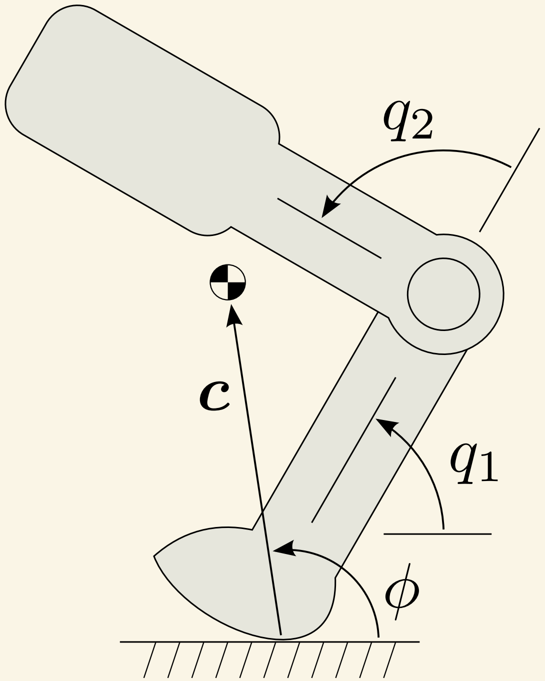
Case 1: Planar Double Pendulum

The control system can control the velocity of the actuated joint (joint 2).

It wants to control the velocity of the CoM via motion of the actuated joint.

In particular, it wants to control either $\dot{\phi}$ or $\dot{c}_x$

Velocity Gain



Case 1: Planar Double Pendulum

So we define an *angular velocity gain*

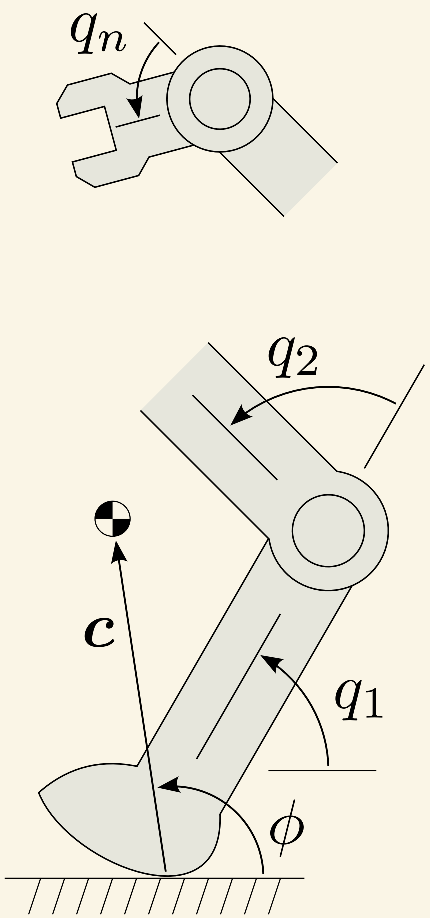
$$G_\omega = \frac{\Delta\dot{\phi}}{\Delta\dot{q}_2}$$

and a *linear velocity gain*

$$G_{\mathrm{v}} = \frac{\Delta\dot{c}_x}{\Delta\dot{q}_2}$$

where $\Delta\dot{\phi}$, $\Delta\dot{c}_x$ and $\Delta\dot{q}_2$ are step changes in the velocities $\dot{\phi}$, $\dot{c}_x$ and $\dot{q}_2$ caused by a nonzero impulse at joint 2.

Velocity Gain

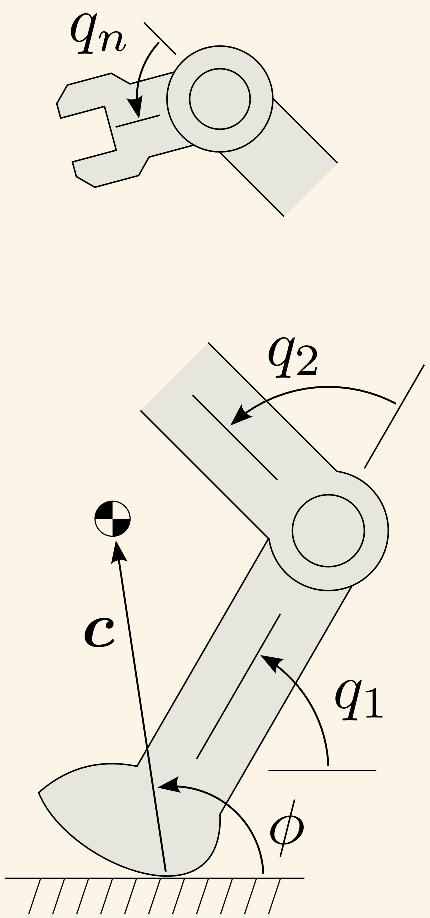


Case 2: General Planar Mechanism

If there is more than one actuated joint then there is a choice of motions to use for balancing.

We therefore define a *virtual joint*, with joint variable q_{v}, which describes the particular motion that will be used for balancing.

Velocity Gain



Case 2: General Planar Mechanism

The virtual joint is mapped to the actuated joints by

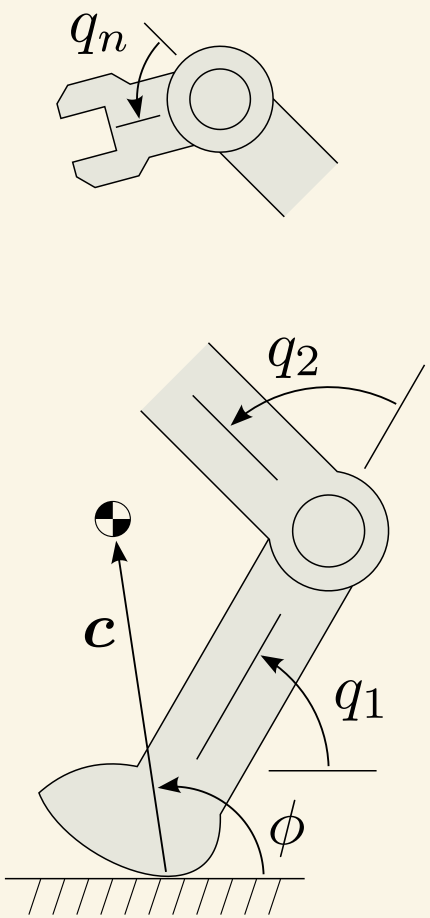
$$q_i = f_i(q_\mathrm{v})\,, \qquad i = 2 \ldots n$$

and

$$\dot{q}_i = \frac{\partial f_i}{\partial q_\mathrm{v}} \dot{q}_\mathrm{v}\,, \qquad i = 2 \ldots n$$

where f_i are functions chosen by the user.

Velocity Gain



Case 2: General Planar Mechanism

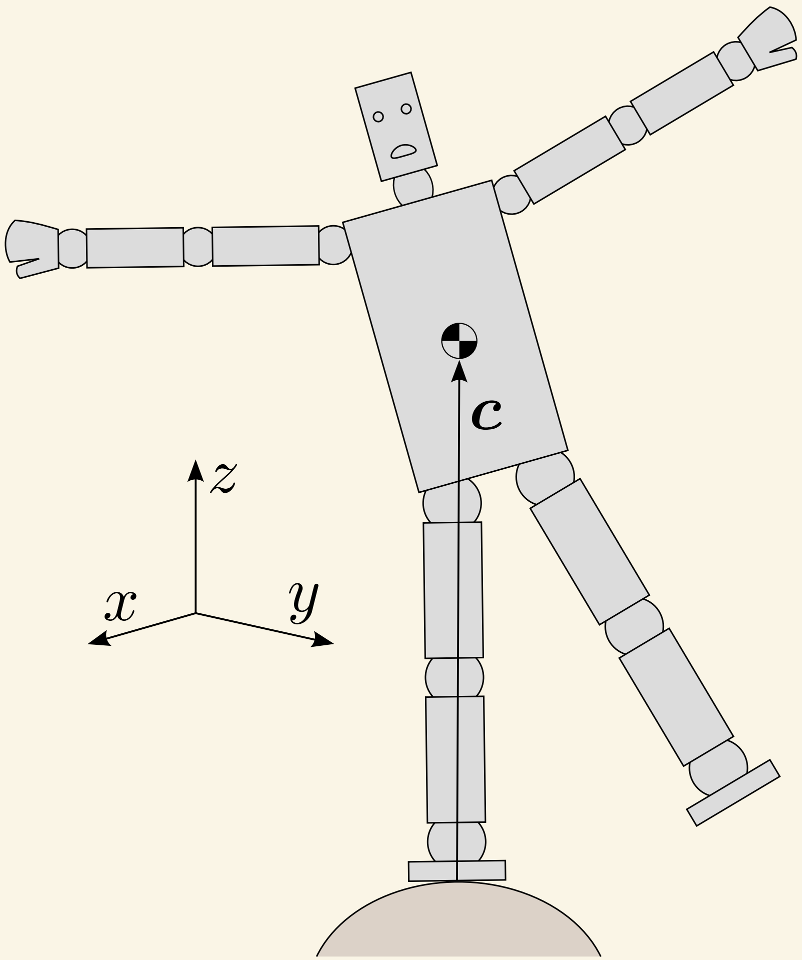
The velocity gains are now

$$G_\omega = \frac{\Delta\dot{\phi}}{\Delta\dot{q}_\mathrm{v}} \quad \text{and} \quad G_\mathrm{v} = \frac{\Delta\dot{c}_x}{\Delta\dot{q}_\mathrm{v}}$$

where $\Delta\dot{\phi}$ and $\Delta\dot{c}_x$ are step changes in the velocities $\dot{\phi}$ and $\dot{c}_x$ resulting from an *impulse vector* that causes the step change $\Delta\dot{\boldsymbol{q}}_\mathrm{a}$ in the actuated joint velocities, given by

$$\Delta\dot{\boldsymbol{q}}_\mathrm{a} = \begin{bmatrix} \frac{\partial f_2}{\partial q_\mathrm{v}} & \frac{\partial f_3}{\partial q_\mathrm{v}} & \cdots & \frac{\partial f_n}{\partial q_\mathrm{v}} \end{bmatrix} \Delta\dot{q}_\mathrm{v}$$

Velocity Gain



Case 3: General Spatial Mechanism

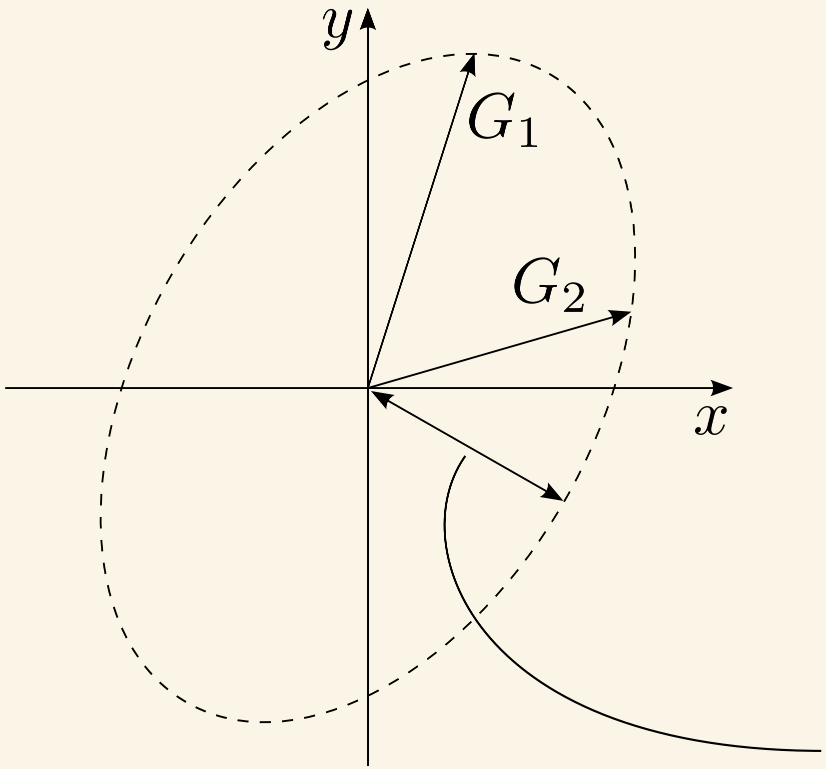
In 3D the velocity gains are vectors:

$$G_\omega = \frac{\boldsymbol{c} \times \Delta\dot{\boldsymbol{c}}}{|\boldsymbol{c}|^2} \quad \text{and} \quad G_\text{v} = \begin{bmatrix} \Delta\dot{c}_x \\ \Delta\dot{c}_y \end{bmatrix}$$

(3D vector) (2D vector)

where $\Delta\dot{\boldsymbol{c}}$ is the step change in CoM velocity resulting from the impulse corresponding to $\Delta\dot{q}_\text{v} = 1$

Velocity Gain



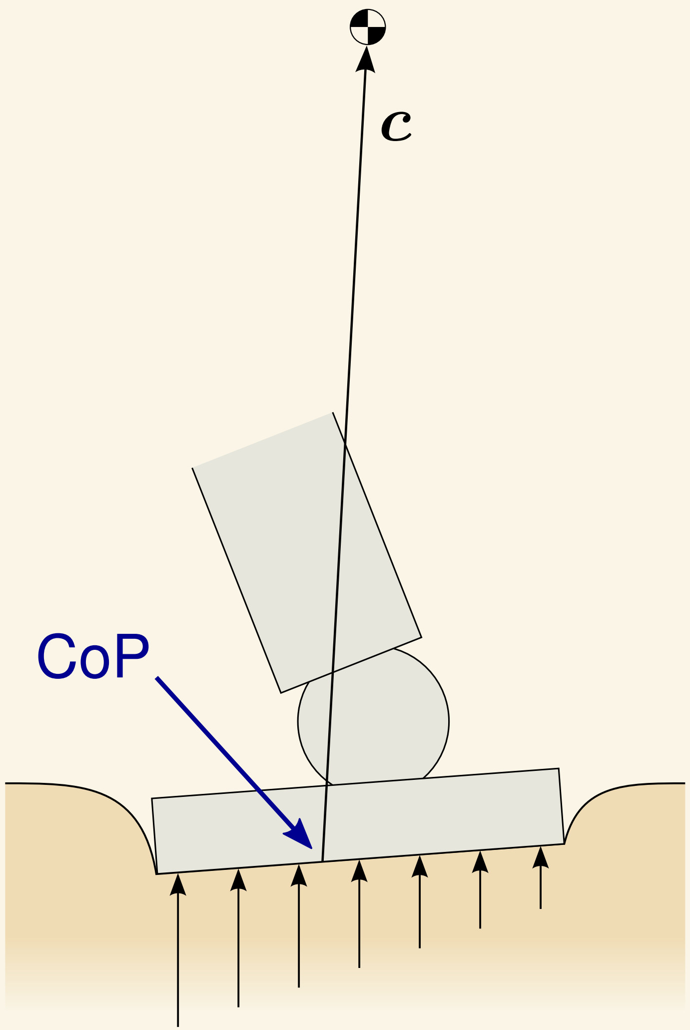
Case 3: General Spatial Mechanism

In 3D one must devote *two* virtual joints to the task of balancing, each with its own velocity gain.

Together, these gains define an ellipse (if one is using a 2-norm) showing how the robot's ability to balance varies with direction.

worst case: direction in which the robot is least able to balance

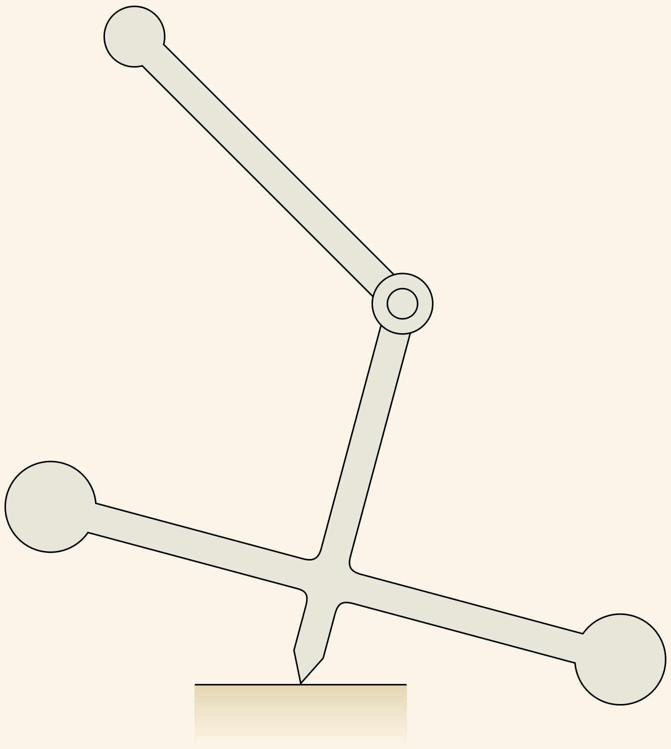
Area Contact



If the robot is standing on hard ground then balancing is trivial – even a statue can do it.

But if the robot is standing on soft ground then the *centre of pressure* can be used in place of the contact point to define the velocity gains.

Examples



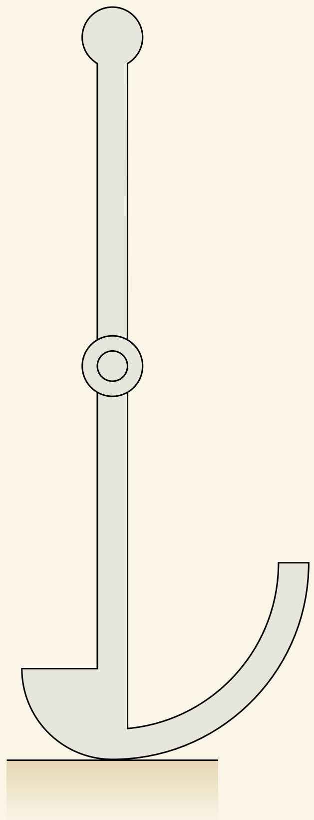
The 'impossible' balancer

It is physically impossible to balance this mechanism because $G_\omega = 0$ in every configuration.

There are infinitely many mechanisms with this property.

(from Featherstone 2013)

Examples

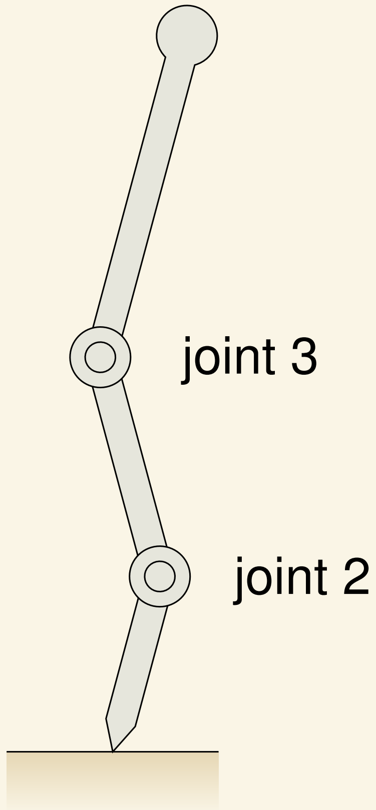


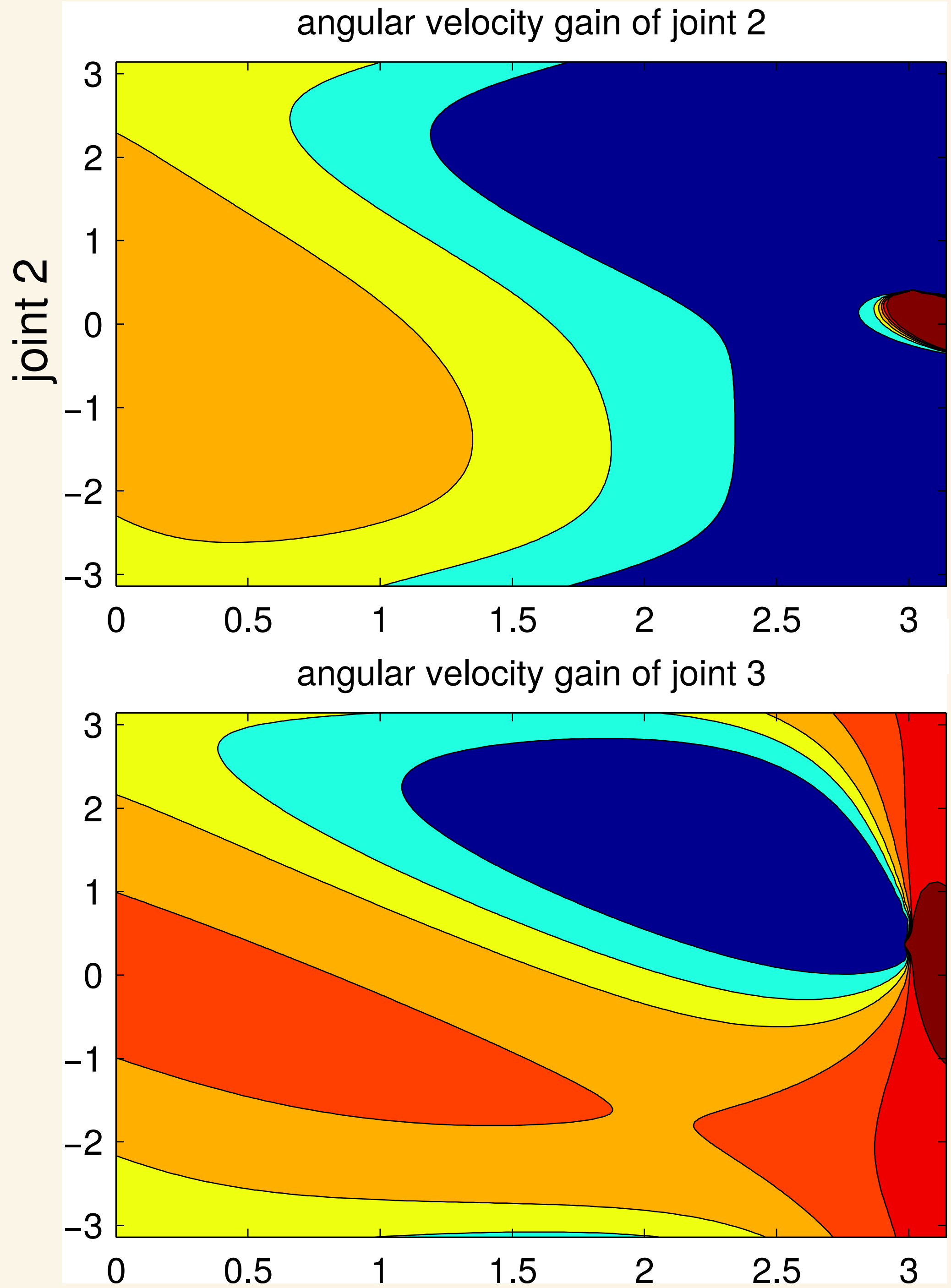
This mechanism has a foot composed of two circular arcs. There is a step-change in curvature where the two arcs meet. $G_\omega > 0$ along one arc and $G_\omega < 0$ along the other.

The robot cannot balance at the transition point (shown) because it has two ways to correct a balance error in one direction and no way to correct a balance error in the other.

(from Featherstone 2013)

Velocity Gain Maps





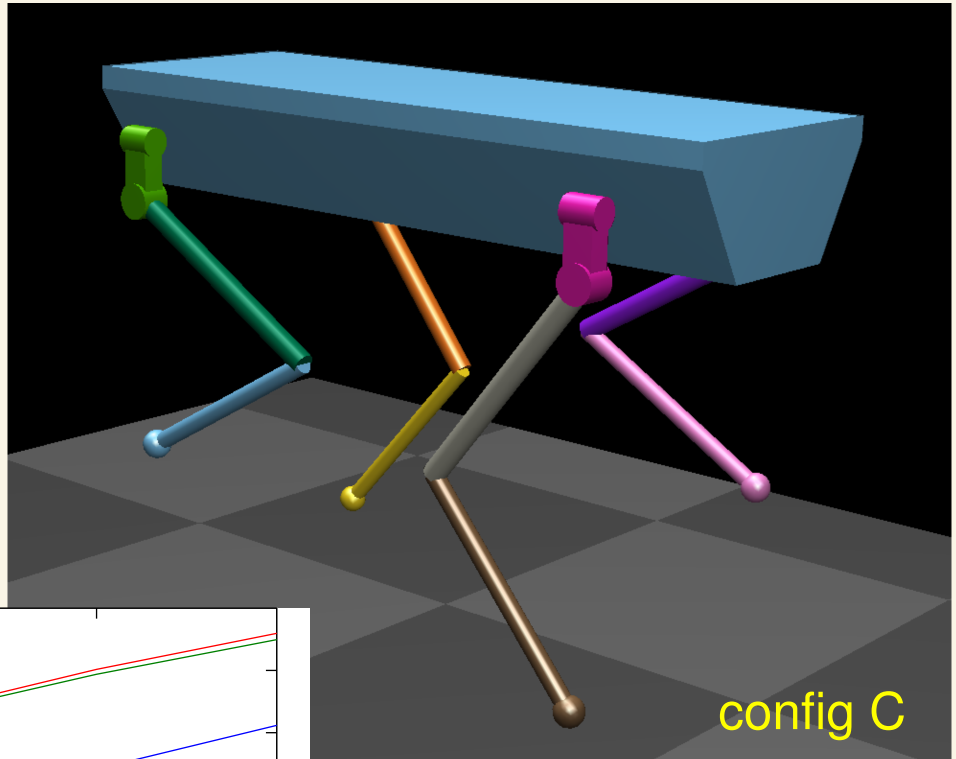
reveal where the robot is good and bad at balancing, and which joints to use.

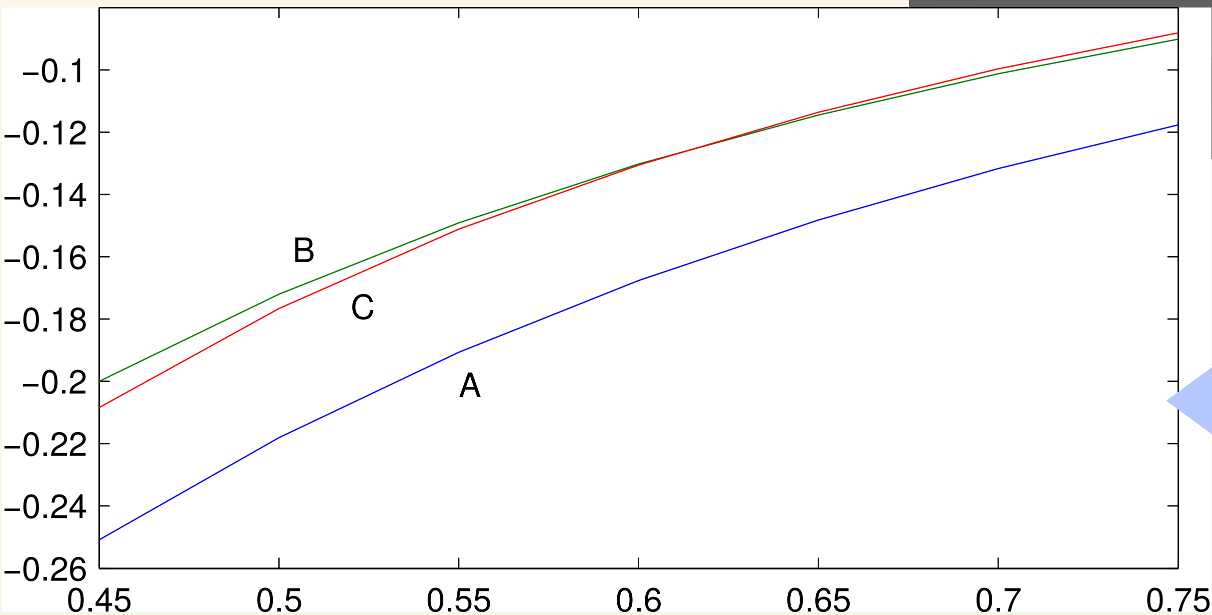
Analysis of Existing Robots

How well can HyQ balance on two feet?

What are the best configurations for balancing?

What is a good virtual joint?





G_ω vs. torso height for various positions of the free legs

Conclusion

- Velocity gains provide *quantitative measures* of a robot's *physical ability balance*.
- They can be used to design new robots, and to analyse existing ones.

[a future talk]

- They can also be used to describe the physics of balancing, and to implement balance control systems.