The Ring Screw Mechanism

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Abstract

This document presents a technical description of the ring screw mechanism, which is a device that provides a nearly frictionless screwing motion between a screw rod and a nut. It therefore provides the same functionality as a ball screw, but offers certain advantages, including the ability to operate at much higher speeds. The ring screw is the subject of patent application number PCT/IB2016/052739, with priority date in May 2015, and much of the content of this document repeats (with better explanations) the description that appeared originally in that patent application.

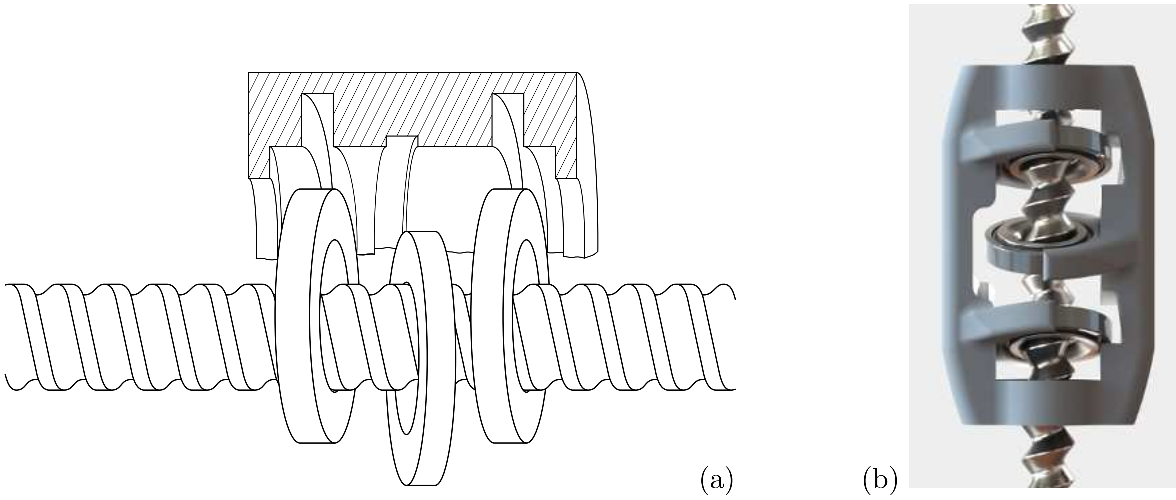
1 Overview

A ring screw mechanism is a mechanical device that achieves a theoretically perfect rolling contact between a screw rod and a nut. It does this by means of a set of three or more ball bearings that are fixed in particular places within the nut. The inner rings of these bearings make contact with the rod as it is screwed into the nut; and once the rod has been screwed sufficiently far into the nut, so that it has made contact with a sufficient number of rings, these contacts constrain the rod so that the only motion it can make is a screwing motion relative to the nut. The shape and placement of the rings, together with the shape of the screw thread in the rod, are such that each ring makes a line contact with the rod, and rolls without sliding as the rod is turned.

This perfect rolling line contact is a defining feature of the ring screw mechanism, and distinguishes it from other similar mechanisms that can be found in the patent literature, of which [1, 3, 5, 6] is just a small sample. Compared with these earlier devices, a ring screw is more efficient and can deliver larger thrust forces per contact. (Tests on our first functional prototype indicate 90% efficiency [4].)

A ring screw therefore provides the same functionality as a ball screw. Its main advantage over a ball screw is that it can operate at much higher speeds. At the time of writing, our first functional prototype has been operated at speeds of up to 16,500rpm [4]. The maximum speed of a ring screw is determined either by the maximum speed of the bearings, or, more likely, the resonant frequency of the rod. If the rod is short then the maximum reachable speed may be limited by the maximum acceleration of the motor.

Figure 1(a) shows an example of a ring screw mechanism in which most of the nut's frame has been cut away, revealing the rings inside; and what remains of the frame has been drawn separated from the rings so as to show the slots that hold the bearings in their correct places. The rings themselves are drawn in approximately their correct positions, but with exaggerated offsets, so that it can be seen how the rings are both tilted and offset relative to the rod.



(a) (b)

Figure 1: Ring screw mechanism sketch (a) and CAD rendering (b), the latter by E. Heijmink

The CAD rendering in Figure 1(b) provides a more accurate picture of a complete ring screw mechanism.

The primary purpose of this document is to provide the equations that determine the correct amount of tilt and offset of the rings, as well as the correct shape of the inner surfaces of the rings and the surface of the rod, so that each ring makes a theoretically perfect rolling line contact with the rod. The reason to aim for a perfect rolling contact is that it maximizes the energy efficiency of the ring screw; and the reason why it is important to make line contact is to maximize the thrust force that the ring screw can produce. As can be seen, there are relatively few contacts between the rod and the nut, so each one must be able to transmit a substantial force.

One topic that is not covered in this document is the design of the nut's frame. Ideally, the frame should be designed so that the rings contribute equally to the production of thrust forces, in spite of small imperfections in the manufacture of the parts. This suggests that a degree of compliance be present somewhere in the nut. Another idea is that the frame should be made deliberately the wrong shape, so that the distortion of the frame as the rod is screwed into the nut causes each ring to apply the correct preload force to the rod. We have designed nuts with these properties already, but this topic is still a work in progress.

2 Finding the Lines

The ring screw exploits a basic result from a branch of mathematics called *screw theory*, which says that the sum of two pure rotations is, in general, a screwing motion. In the case of the ring screw, one of these rotations is the rotation of a ring within its bearing; the other is the pure rolling contact between this ring and the screw rod;[1] and the sum of these two rotations is the screwing motion of the rod relative to the nut. However, it is not necessary to understand screw theory in order to follow the equations in this section, as we shall be using nothing more complicated than ordinary 3D vectors.

[1] If a rigid body B_1 is rolling without slipping over a body B_2 then it is necessarily the case that the motion of B_1 relative to B_2 at any given instant is a pure rotation. This is because B_1 is touching B_2 at at least one point, and this point (in B_1) must have zero velocity relative to B_2. Any instantaneous motion of a rigid body in which at least one point in that body has zero velocity is necessarily a pure rotation about a line passing through that point. This line is called the body's *instantaneous rotation axis*. In general, this line moves as the body rolls. However, in the special case of a ring screw, this line has a fixed location in the nut.

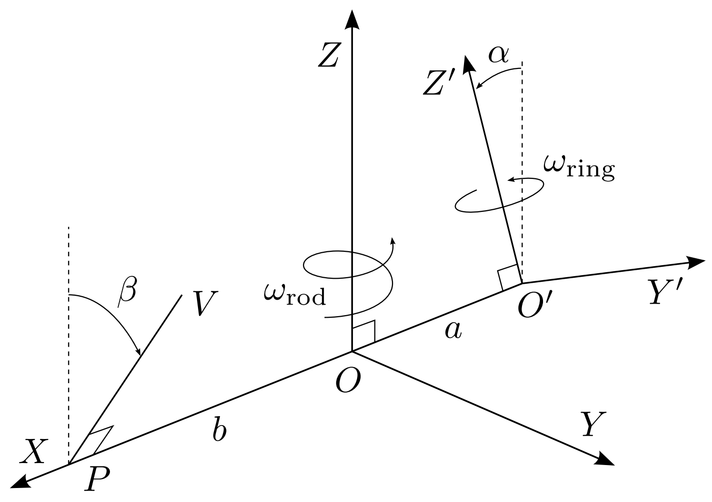


Figure 2: Geometry of perfect rolling contact between a screw rod and a ring

The analysis begins with an investigation of the relationship between the rod and a single ring, taking the nut to be fixed in space. In this scenario, the rod screws along a line that is fixed in space, while the ring rotates about a different line that is also fixed in space, and that is both tilted and offset relative to the screw axis of the rod. We seek a relationship between the motions of these two parts having the property that there exists a line somewhere in space where the velocities of points in the rod exactly equal the velocities of corresponding points in the ring. This is the line of perfect rolling contact. The analysis reveals both the conditions for this line to exist and its location relative to the other two lines.

Figure 2 shows the screw axis of the rod, labelled Z, and the rotation axis of any one ring, labelled Z'. It also shows the common perpendicular[2] between these two lines, labelled X, which passes through Z and Z' at the points O and O'. The figure also shows two Cartesian frames with their origins at O and O' and their z axes aligned with Z and Z'. These frames define two Cartesian coordinate systems such that any point in space has coordinates x, y and z in the frame at O and x', y' and z' in the frame at O'. The location of Z' relative to Z is described by an offset distance, a, and a tilt angle, α, the latter measured in the direction shown in the diagram. All angles in this document are expressed in units of radians. Distances can be expressed in any desired length unit, provided that the same unit is used throughout. Figure 2 also shows a line V, which will be discussed later on.

Let the screw thread on the rod have a pitch of h measured in lead per radian, not lead per revolution. A positive value for h means that the screw is right-handed, and a negative value means that it is left-handed. For now, we shall assume that h is positive. If the rod is rotating with an angular velocity of ω_{rod}, measured in radians per second, then the linear velocity of a point in the rod having coordinates (x, y, z) at the current instant is

$$\boldsymbol{v}_{\text{rod}}(x, y, z) = \omega_{\text{rod}} \begin{bmatrix} -y \\ x \\ h \end{bmatrix} . \tag{1}$$

Likewise, if the ring is rotating at a rate of ω_{ring}, also measured in radians per second, then the

[2] A common perpendicular to two given lines is a line that passes through both lines at right angles. Any two lines in space have a unique common perpendicular, unless they happen to be parallel, in which case they have infinitely many common perpendiculars. Thus, given that $\alpha \neq 0$, there is only one line X that passes through both Z and Z' at right angles as shown in Figure 2.

linear velocity of a point in the ring having coordinates (x', y', z') is

$$\boldsymbol{v}'_{\text{ring}}(x', y', z') = \omega_{\text{ring}} \begin{bmatrix} -y' \\ x' \\ 0 \end{bmatrix}$$

expressed in the coordinate system at O'. Transforming this equation to the coordinate system at O gives

$$\boldsymbol{v}_{\text{ring}}(x, y, z) = \omega_{\text{ring}} \begin{bmatrix} -\sin(\alpha)z - \cos(\alpha)y \\ \cos(\alpha)(x + a) \\ \sin(\alpha)(x + a) \end{bmatrix}. \tag{2}$$

We seek a line in space with the property that $\boldsymbol{v}_{\text{ring}}(x, y, z) = \boldsymbol{v}_{\text{rod}}(x, y, z)$ at every point on the line. This problem can be solved simply by equating the expressions in Eqs. 1 and 2:

$$\frac{\omega_{\text{rod}}}{\omega_{\text{ring}}} \begin{bmatrix} -y \\ x \\ h \end{bmatrix} = \begin{bmatrix} -\sin(\alpha)z - \cos(\alpha)y \\ \cos(\alpha)(x + a) \\ \sin(\alpha)(x + a) \end{bmatrix}. \tag{3}$$

This is a set of three equations in six unknowns: x, y, z, a, α and the speed ratio $\omega_{\text{rod}}/\omega_{\text{ring}}$. The solution to this set of equations is the line labelled V in Figure 2. This line is perpendicular to X and meets it at the point P located at a distance b from O; and it is tilted by an angle β relative to Z, this angle being measured in the direction shown in the diagram, which is opposite to the direction of measurement of α. The four parameters a, α, b and β satisfy the two equations

$$\tan(\alpha) = \frac{h}{b} \qquad \text{and} \qquad \tan(\beta) = \frac{h}{a}, \tag{4}$$

and the speed ratio is

$$\frac{\omega_{\text{rod}}}{\omega_{\text{ring}}} = \frac{a + b}{\sqrt{h^2 + b^2}}. \tag{5}$$

This solution is unique. There is no other line where the velocities match as required.

For a right-handed screw thread, h, α and β are all strictly positive, and this is the situation shown in Figure 2. If the screw thread is left-handed then h, α and β are all strictly negative, but the two distances a and b are still positive. Equations 4 and 5 apply in both cases.

3 Finding the Surfaces

Having found the one line in space where perfect rolling contact is possible, the next step is to design the inner surface of the ring and the outer surface of the rod so that they make contact only along this line. In the case of the ring, the argument proceeds as follows: if the ring rotates about Z', and yet always makes contact with the rod along V, then the surface of the ring must be the surface of revolution that is generated by sweeping V in a circle of radius $a + b$ around Z'. A surface of this kind is a well-known mathematical surface, and is called a *hyperboloid of one sheet*. An example of such a surface is shown in Figure 3(a). The equation describing the inner surface of the ring is

$$x'^2 + y'^2 = (a + b)^2 + \tan(\alpha + \beta)^2 z'^2, \tag{6}$$

where x', y' and z' are coordinates expressed in the coordinate frame located at O', $a + b$ is the radius of the surface at its narrowest point, and $\alpha + \beta$ is the tilt angle of V relative to Z'.

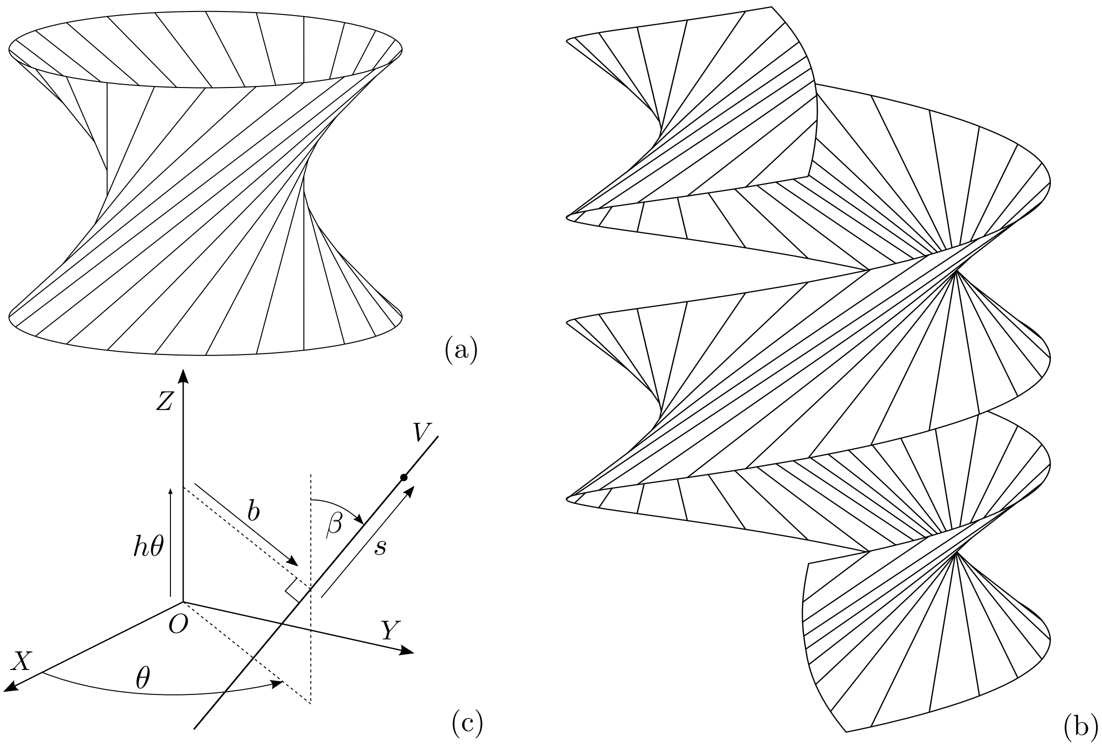


Figure 3: Examples of a hyperboloid (a) and an oblique open ruled generalized helicoid (b); and the definition of the coordinates θ and s used to describe the helicoidal surface of the screw rod (c).

Applying the same reasoning to the rod, we can see that the outer surface of the rod must be the surface that is formed by sweeping the line V around Z in a helix of radius b and pitch h. In this case the resulting surface is not well known and goes by the rather cumbersome name of *oblique open ruled generalized helicoid*, which we shall abbreviate to 'helicoid'. An example of such a surface is shown in Figure 3(b). The equation describing this surface is best expressed in terms of a pair of surface coordinates, θ and s, denoting the sweep angle of V and the location of a point on V, as shown in Figure 3(c). In terms of θ and s, the equation of the surface, expressed in the coordinate frame at O, is

$$\begin{aligned} x &= b\,\cos(\theta) - s\,\sin(\beta)\,\sin(\theta) \\ y &= b\,\sin(\theta) + s\,\sin(\beta)\,\cos(\theta) \\ z &= h\,\theta + s\,\cos(\beta)\,. \end{aligned} \tag{7}$$

Equations 6 and 7 show us that the shape of the rod's surface is a function of the three parameters b, β and h, whereas the shape of the ring's inner surface is a function of only two parameters: $a + b$ and $\alpha + \beta$. If we take into account also Eq. 4 then we obtain the following results:

1. For any one given rod shape, there is exactly one corresponding ring shape.
2. For any one given ring shape, there is a one-parameter family of rod shapes that will work with this ring.

It therefore follows that every ring in a ring screw mechanism must have the same shape, and also the same offset and tilt angle. They can differ only in their placement around the helix of the rod. (The rings in Figure 1, for example, are placed two and a half turns apart.)

An important feature of Eq. 6 is that if $\alpha + \beta = \pi/2$ (or $-\pi/2$ in the case of a left-handed screw) then the hyperboloid surface collapses to a plane with a hole in the middle, and the thickness of the corresponding part of the ring drops to zero. Furthermore, if we consider the possibility of $\alpha + \beta > \pi/2$ (or $< -\pi/2$ for a left-handed screw) then we find that the ring and rod interfere. So any physical implementation of a ring screw must have $|\alpha + \beta| < \pi/2$. This restriction on the values of the angles implies (via Eq. 4 and the trigonometric formula for the tangent of the sum of two angles) that $ab > h^2$, and this in turn implies $\omega_{\text{rod}}/\omega_{\text{ring}} > 1$. So

$$|\alpha + \beta| < \frac{\pi}{2}, \quad ab > h^2 \quad \text{and} \quad \frac{\omega_{\text{rod}}}{\omega_{\text{ring}}} > 1\,. \tag{8}$$

4 Completing the Design

Having established both the conditions for perfect rolling contact and the mathematical surfaces of the screw thread and the inner surface of the ring, all that remains are a few practical details, such as the outer radius of the screw rod and the placement of the rings around it. We now consider these items.

4.1 Cutting the Ring

Figure 4(a) shows the precise location of the ring and the line of perfect rolling contact relative to the screw rod. In relation to Figure 2, this figure shows a view looking towards O from a point on the X axis beyond P. For clarity, only the ring itself is shown, not the ball bearing on which it is mounted. Three essential and highly distinctive features of the ring screw mechanism are evident in this figure:

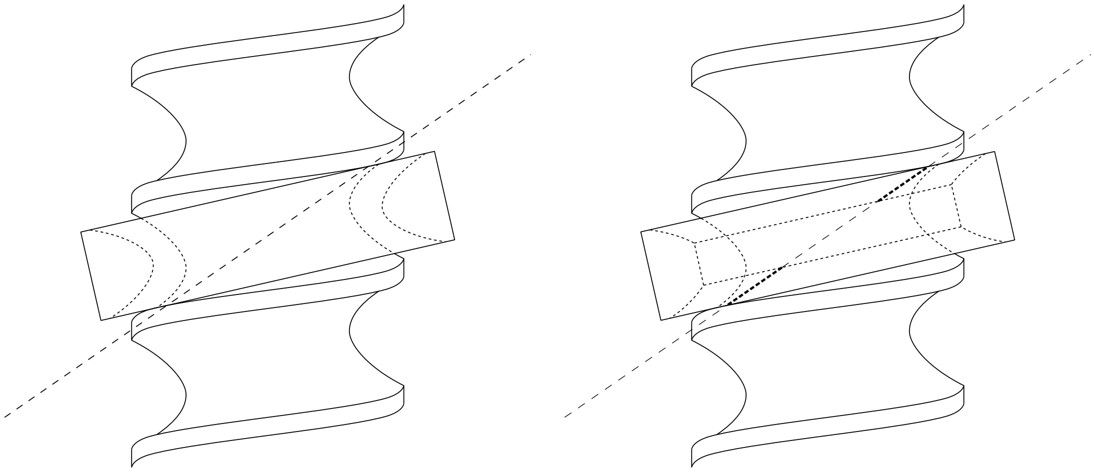
1. the ring is narrower than the groove into which it fits;
2. the ring is tilted at a greater angle than the helix angle of the screw thread at the rod's outer diameter; and
3. the line of perfect rolling contact is tilted at an even greater angle than the ring.

These features follow inevitably from the design requirement of a perfect rolling line contact. Although pictures of mechanisms that resemble the ring screw can be found in the patent literature, none have these features; and from their absence one can immediately conclude that these earlier inventions do not exhibit perfect rolling line contact.

The line V in Figure 2 extends to infinity in both directions; but a physical ring can make contact with a physical rod along only a finite portion of this line. One more observation that can be made from Figure 4(a) is that the length of this finite portion can be quite substantial. In fact, it can be longer than the outer diameter of the rod.

Although Figure 4(a) does not show any contact normals, one can see from the shapes of the contacting surfaces that the normals near the top of the contact line point diagonally up,[3] those near the bottom point diagonally down, and those near the middle point approximately straight into the page. Thus, the ring can transmit an upward thrust force to the rod via the upper portion of the contact line, and a downward thrust force via the lower portion of the contact line, but the middle portion contributes very little to the production of thrust forces. So it is possible to cut away the innermost portion of the ring, as shown in Figure 4(b), without significantly reducing the ring's ability to transmit a thrust force to the rod.

[3] As we are discussing here the forces that can be transmitted *from* the ring *to* the rod, the contact normals are defined to be pointing out of the ring and into the rod.



(a) (b)

Figure 4: Relative locations of screw rod, ring and line of perfect rolling contact (a), and cut-away ring making contact along two segments of the line of perfect rolling contact (b)

The reason why one would want to do this is that a lot of useful combinations of the design parameters a, α, b, β and h lead to designs in which the ring is too small to clear the ridge between adjacent grooves on the side of the screw rod opposite to line V. One way to solve this problem would be to lower the height of the ridge by reducing the outer diameter of the rod; but that would reduce the ring screw's ability to deliver a thrust force. The alternative is to cut away the innermost portion of the ring, which has little effect on the thrust force, and is therefore preferable.

4.2 Inner and Outer Radii

Figure 5(a) shows a section through the centre line of the screw rod, labelled with key dimensions, and Figure 5(b) shows a section through the centre line of the ring, also labelled with key dimensions. The shape of the rod depends on the shape of the helicoid, which is a function of the parameters h, b and β, and also on the choice of inner and outer radii, r_1 and r_2. The quantities c and d are the radius at which the helicoidal surface self-intersects, and the distance between adjacent grooves. Clearly, we must have $r_1 \geq b$, $r_1 < r_2$, $r_2 \leq c$ and $d \geq 0$. However, the manufacturing process may require d to be greater than some minimum value if the rod needs to be supported while it is being ground, which imposes a more restrictive upper limit on r_2. The shape of the ring is likewise determined by the shape of the hyperboloid, which depends on the parameters $a + b$ and $\alpha + \beta$, and the inner radius, r_3. A second radius, r_4 (not shown), marks the outer limit of contact with the rod. Beyond r_4, the ring can have any shape, provided that it does not interfere with other parts of the mechanism, such as the rod, the frame or adjacent rings.

4.3 Design Procedure

The objective of the design procedure is to select values for the parameters a, α, b, β, r_1, r_2, r_3 and r_4. Other aspects of the design, such as the length of the screw rod or the shape of the nut frame, are not considered in this document. The number and placement of rings is the topic of the next subsection. There is more than one way to design a ring screw, and the method that we are currently using proceeds as follows.

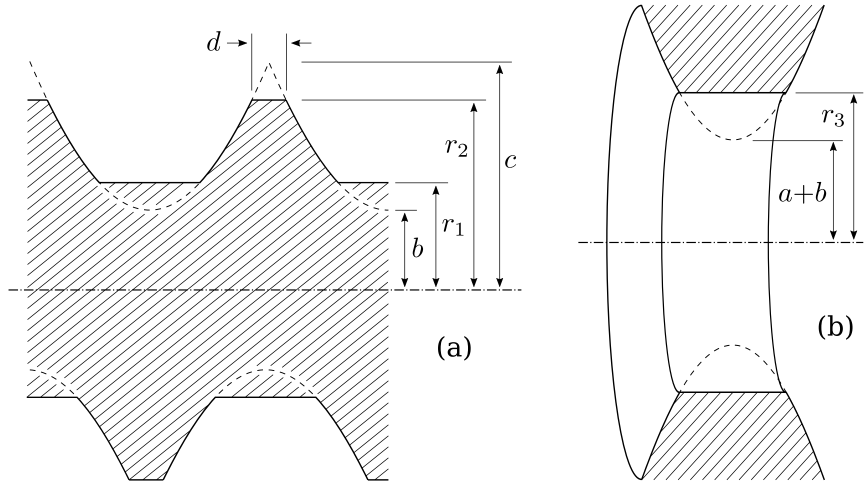


Figure 5: Screw rod and ring dimensions

1. Choose h, r_2, b, d, and the minimum clearance between non-contacting surfaces. h and r_2 are basic functional parameters of a ring screw mechanism, whereas b is a free parameter that can be used to optimize some aspect of the mechanism or its performance, and d may be constrained by the manufacturing process. Smaller values for d allow the rings to be thicker, and therefore easier to make, and may allow them to make contact with the rod along longer line segments. So a good choice for d is at or near the minimum value required by the manufacturing process.

2. Having chosen these quantities, it becomes possible to calculate a, α, β and r_4 using various formulae that can be deduced from the information in this document.

3. The next step is to check for interference between the ring and the rod, and to choose r_3 to be the smallest radius such that the ring clears the rod with the chosen minimum clearance. There is no formula for this, so it must be done numerically. For many combinations of the design parameters, the ring will not interfere with the rod, so no cut-away is needed. In these cases, $r_3 = a + b$.

4. If $r_3 = a+b$ then $r_1 = b$; but if $r_3 > a+b$ then there is the possibility to fill in the bottom part of the groove by choosing a value of $r_1 > b$. This is because if $r_3 > a + b$ then the ring does not reach the bottom of the groove. The trade-off here is between weight and stiffness: filling in the groove increases the weight of the rod, which bad, but also increases its stiffness, which allows both a higher operating speed, because of the higher resonant frequency, and higher thrust forces, because it can cope with larger preload forces.

In general, smaller values of b lead to better angles in the contact normals, resulting in less bending of the rod by the preload forces; but they can also reduce the lengths of the contact line segments, which increases the local stresses near the contact line in both the rod and the rings.

4.4 Number and Location of Rings

All of the rings in one nut have the same offset a, the same tilt angle α and the same hyperboloid inner surface. However, each ring is placed at a different point along the helical path of the screw thread. Any arrangement that sufficiently constrains the motion of the rod will result in a functioning mechanism, but it is preferable for the rings to be as close together as possible in order to minimize the bending moment on the rod caused by the preload forces.

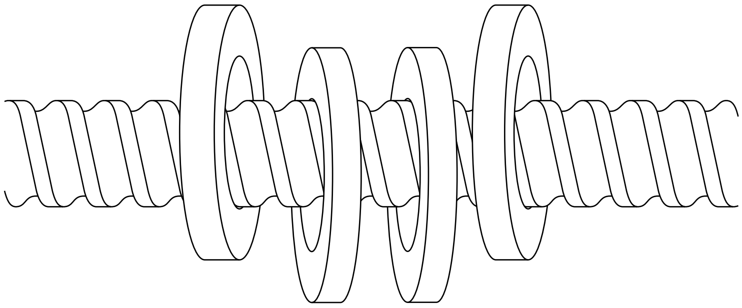


Figure 6: Example arrangement of four rings in a nut

The minimum number of rings is three, and the simplest arrangement of those rings is the one shown in Figure 1, in which the rings are placed $n + \frac{1}{2}$ turns apart, where n is the smallest integer that will work. In the case of Figure 1, $n = 2$. A certain amount of variation is possible. For example, the rings could be placed 2.4 turns apart, or 2.6 turns apart, and it is not necessary for the rings to be equally spaced. However, arrangements such as these should be checked carefully because they do not all result in working ring screw mechanisms. Increasing the number of rings increases the size and weight of the nut, but allows greater thrust forces to be produced, and increases the designer's freedom to choose where the rings are placed.

One obvious weakness of the arrangement in Figure 1 is that the middle ring experiences forces that are twice as large as those on the other two rings. A simple solution to this problem is the four-ring arrangement shown in Figure 6.

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