

High Performance Balancing on a Narrow Support

by Roy Featherstone

presented at the ICRA 2021 Workshop on
High Dynamic Motion Generation
for Under-Actuated Robots



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First view this video:

EITHER

download the shared file [icra2021rf.mp4](#) (if it exists) from the zoom chat panel (if you are using zoom) and view it

OR

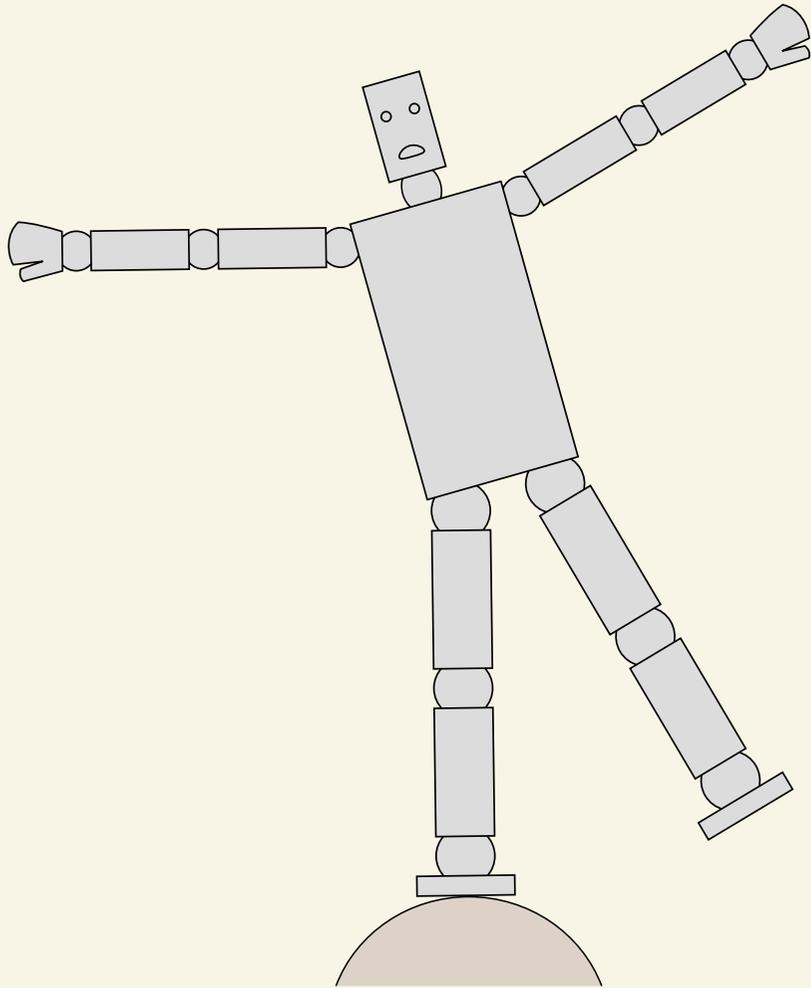
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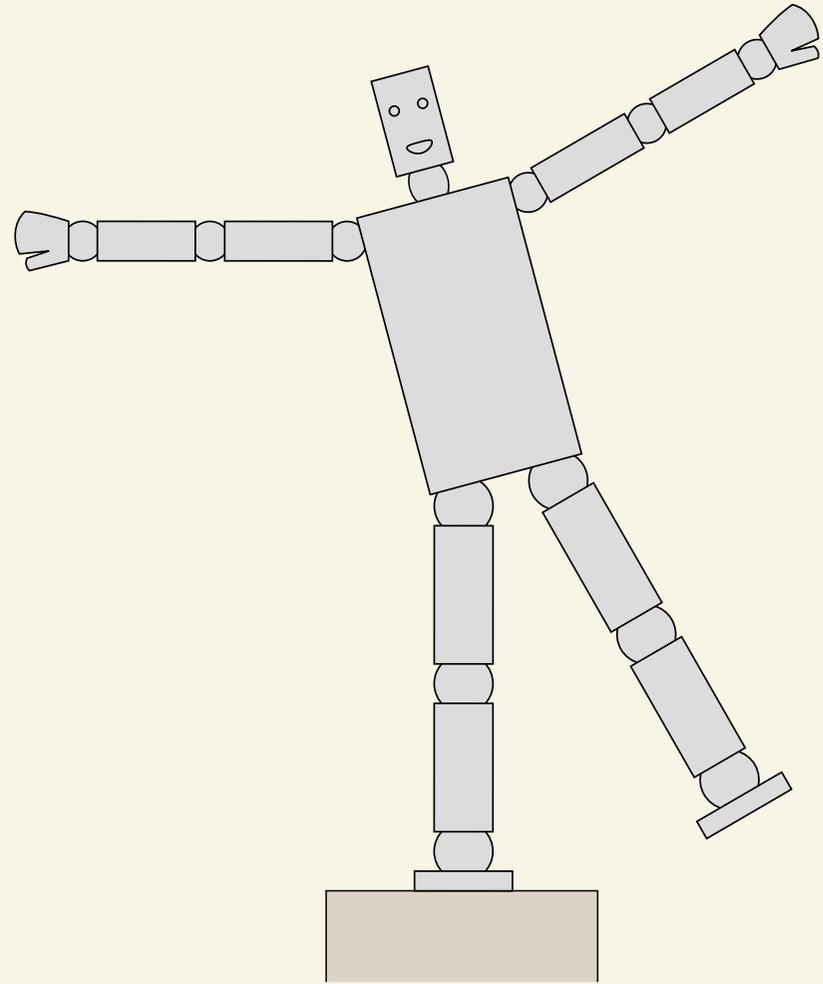
Then return to this slide show.

Waiting for people to return.

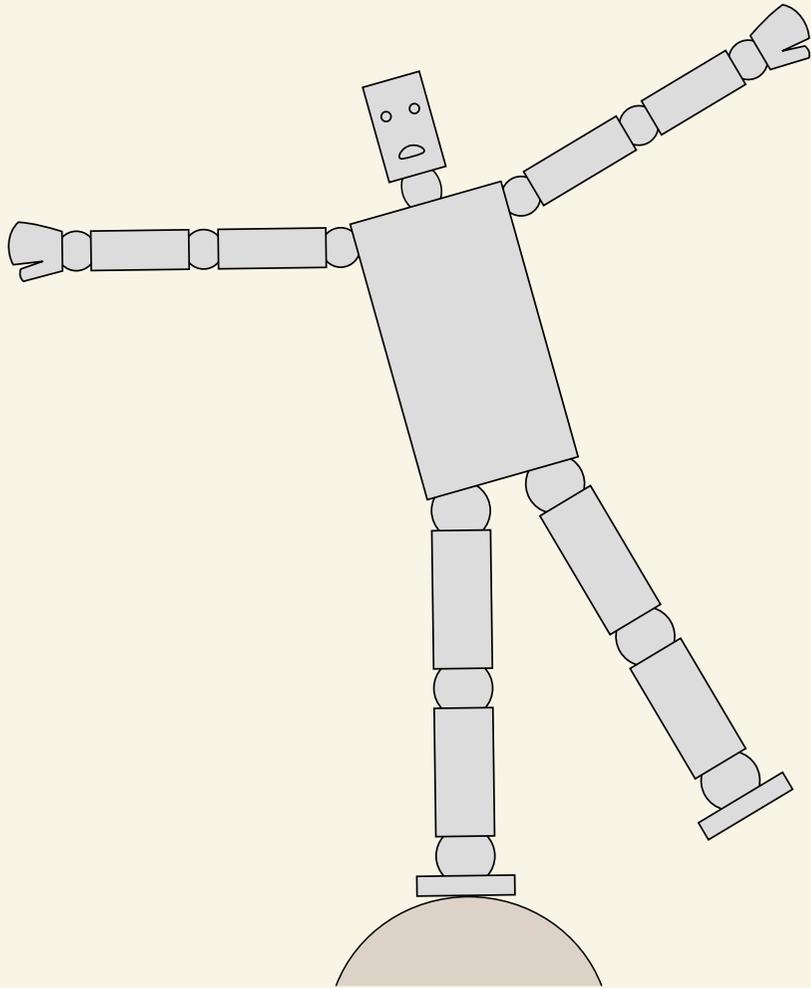
The talk will continue soon.



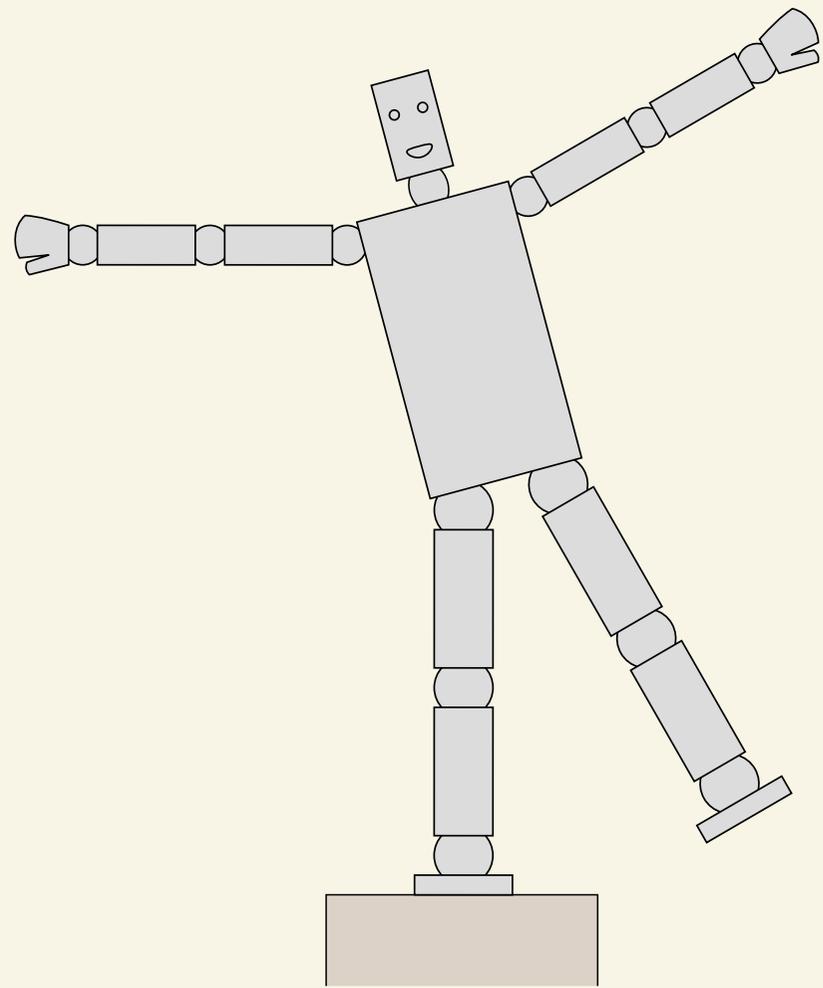
narrow support —
effectively a single point



substantial polygon
of support



dynamically unstable —
must balance actively



statically stable as long as
ground reaction force stays
inside support polygon

High-Performance Balancing

Definition:

A robot is a high-performance balancer if it can

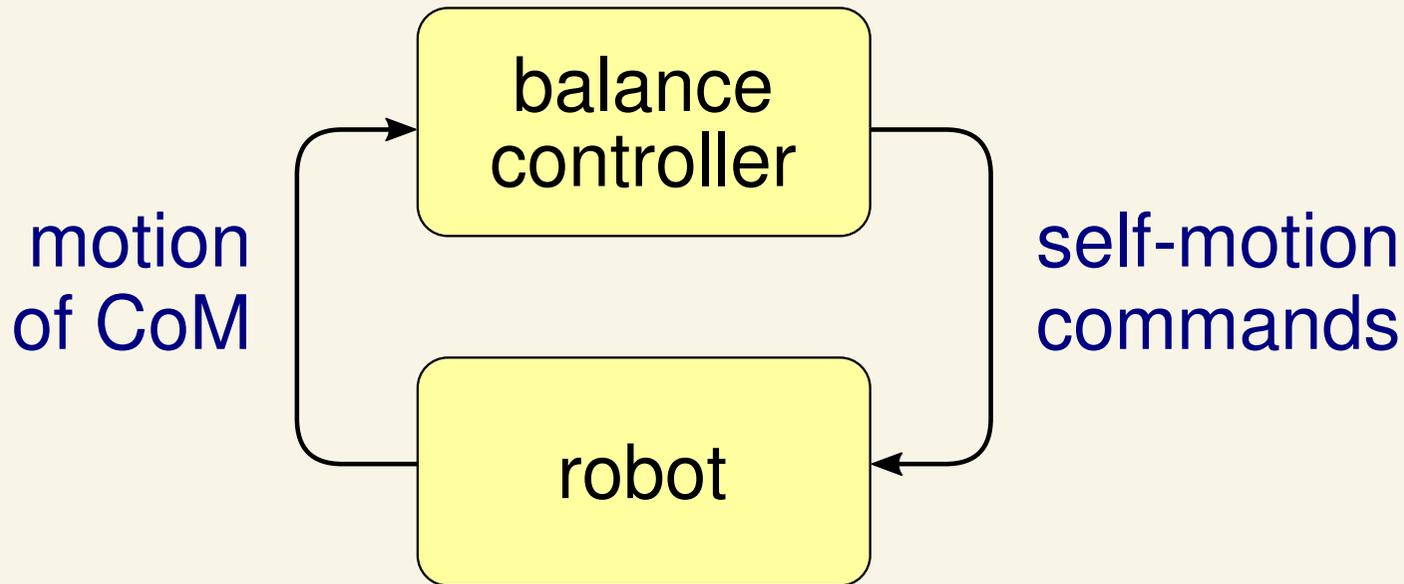
- accurately follow commands to make large, fast movements without losing its balance; and
- quickly recover from large balance disturbances

High-Performance Balancing

Our Approach:

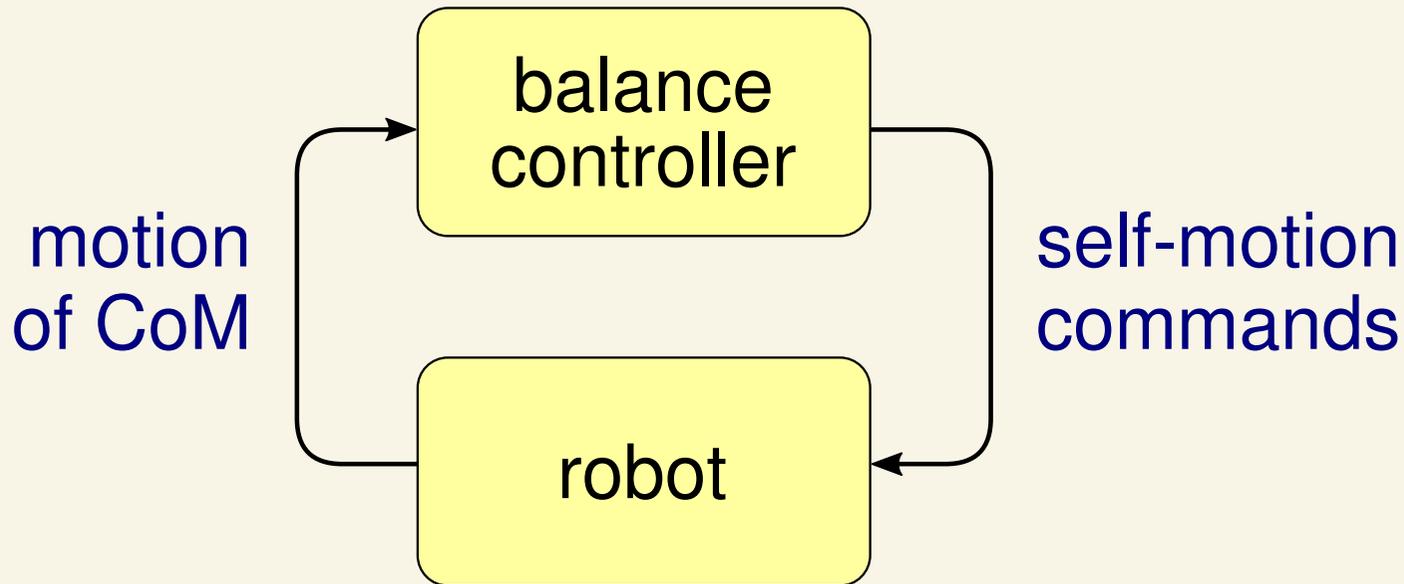
1. Balancing is a physical activity, not just a control theory exercise;
 - so study the *physics of balancing*
2. A robot's physical ability to balance is a property of the robot itself, not the control system;
 - so study what makes a robot good at balancing, and make your robot *good by design*.

Physical Ability to Balance on a Point



Balancing on a point requires moving the robot's centre of mass (CoM) relative to the support point by means of self motions (i.e., motions of the actuated joints).

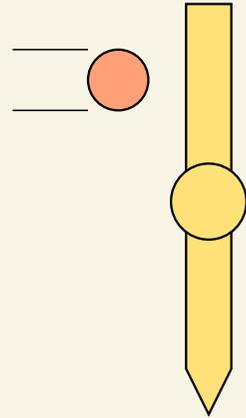
Physical Ability to Balance on a Point



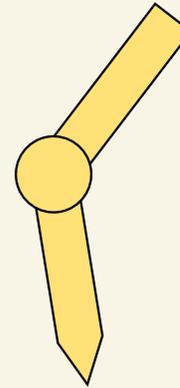
So a robot is *physically good at balancing* if a relatively small self motion causes a relatively large CoM motion.

Physical Ability to Balance on a Point

good balancer



disturbance

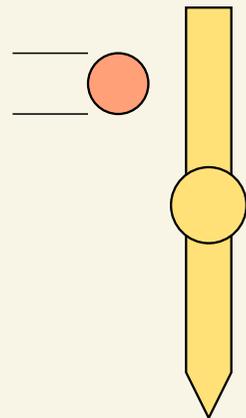


response

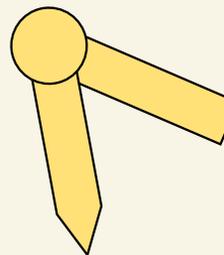


recovery

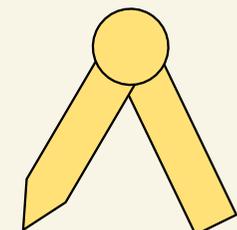
bad balancer



disturbance



response hits motion limit



fall

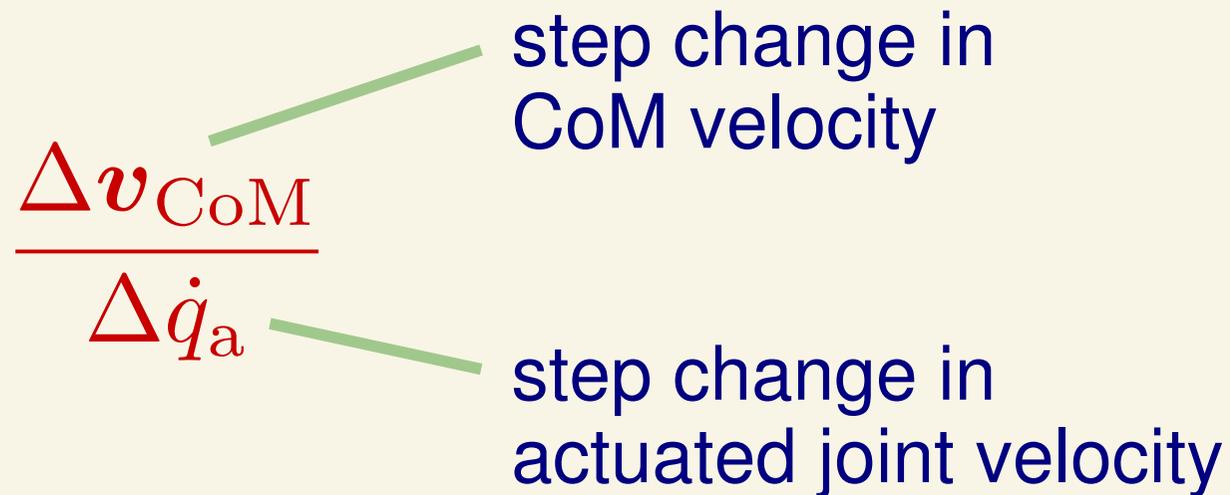
Physical Ability to Balance on a Point

This leads to the idea of *velocity gain*:

$$\frac{\Delta \mathbf{v}_{\text{CoM}}}{\Delta \dot{q}_a}$$

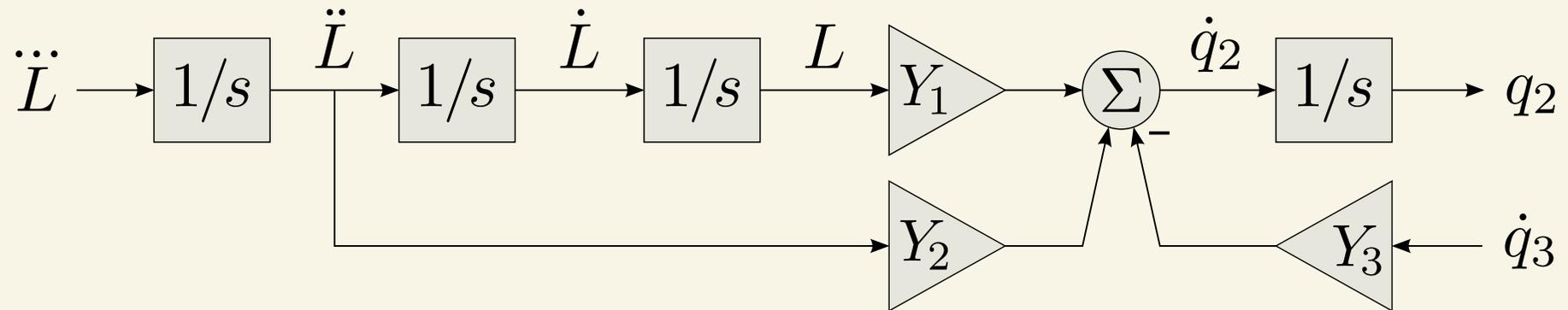
step change in CoM velocity

step change in actuated joint velocity



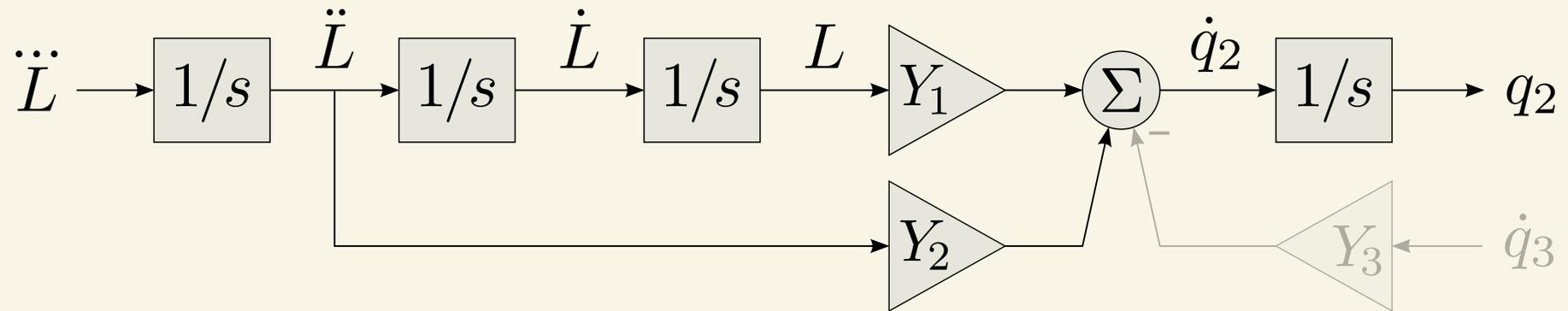
both caused by an impulse at the actuated joint

The Simple Dynamics of Balancing



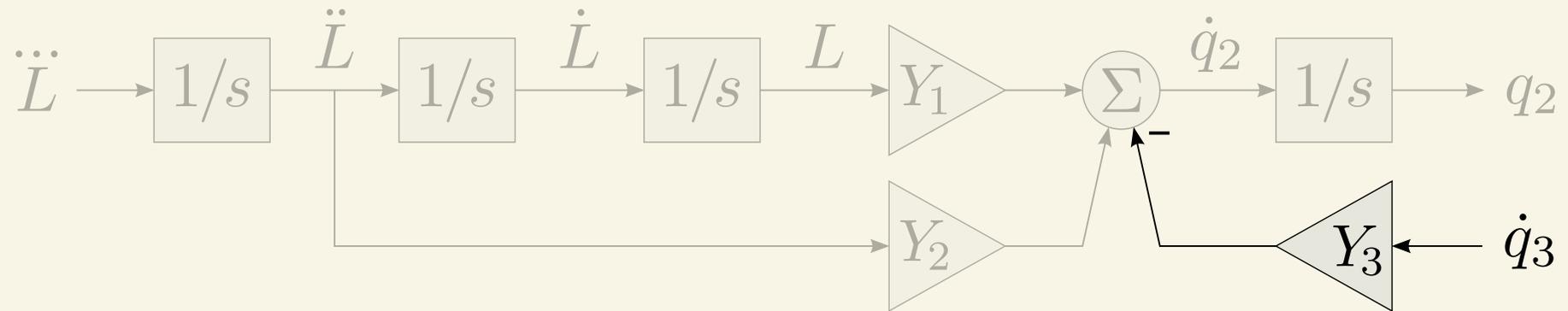
The physical process of balancing can be expressed in the form of this block diagram, which serves as the *plant* for the balance controller to control.

The Simple Dynamics of Balancing



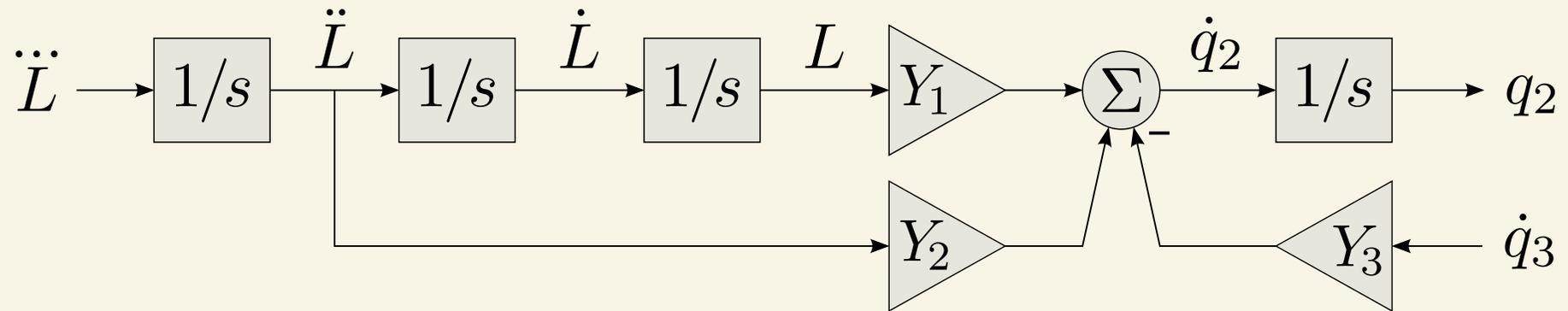
This portion is concerned with balancing

The Simple Dynamics of Balancing



and this portion describes
the disturbances caused by
other motions.

The Simple Dynamics of Balancing



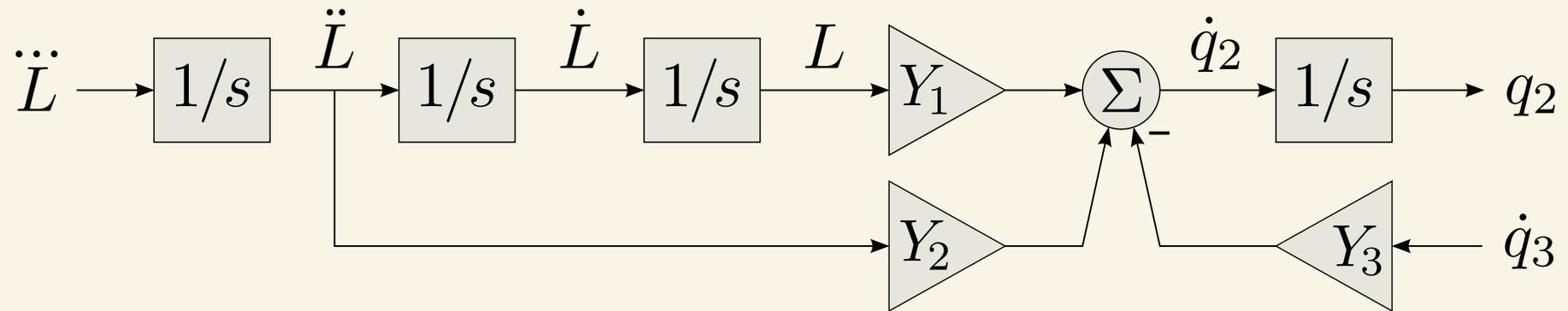
L angular momentum of the robot about the support

q_1 variable(s) describing the robot's motion relative to the ground

q_2 *overloaded* variable(s) used *both* to balance *and* to follow motion commands

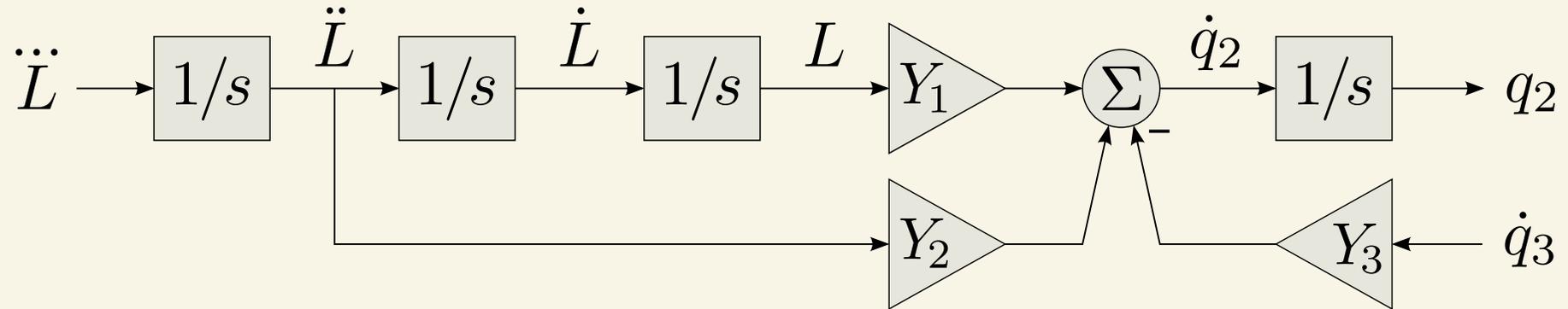
q_3 all other robot variables

The Simple Dynamics of Balancing



The balancing behaviour of the robot depends only on the two gains Y_1 and Y_2 which are easily calculated functions of the robot's configuration variables.

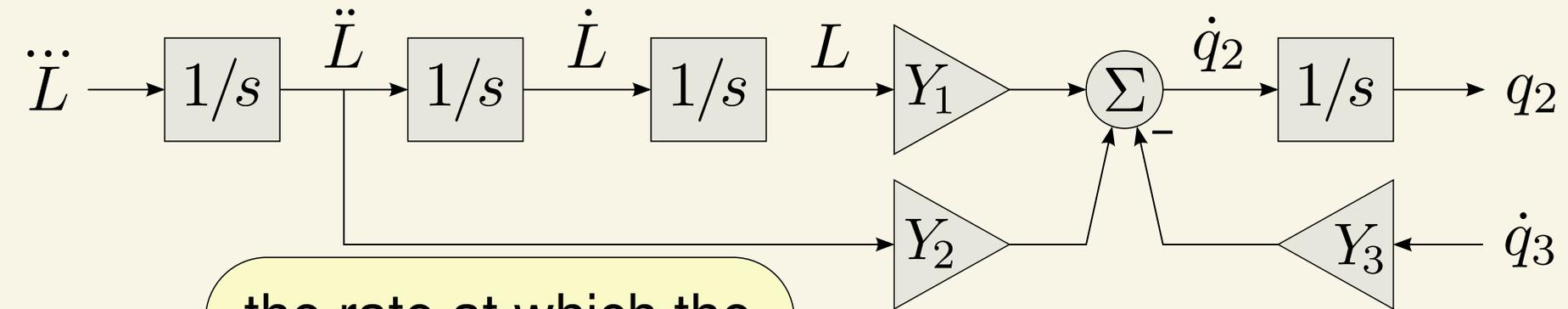
The Simple Dynamics of Balancing



Y_1 and Y_2 are functions of two easily measured physical properties of the robot:

- the natural time constant of toppling, and
- the linear velocity gain of \dot{q}_2

The Simple Dynamics of Balancing



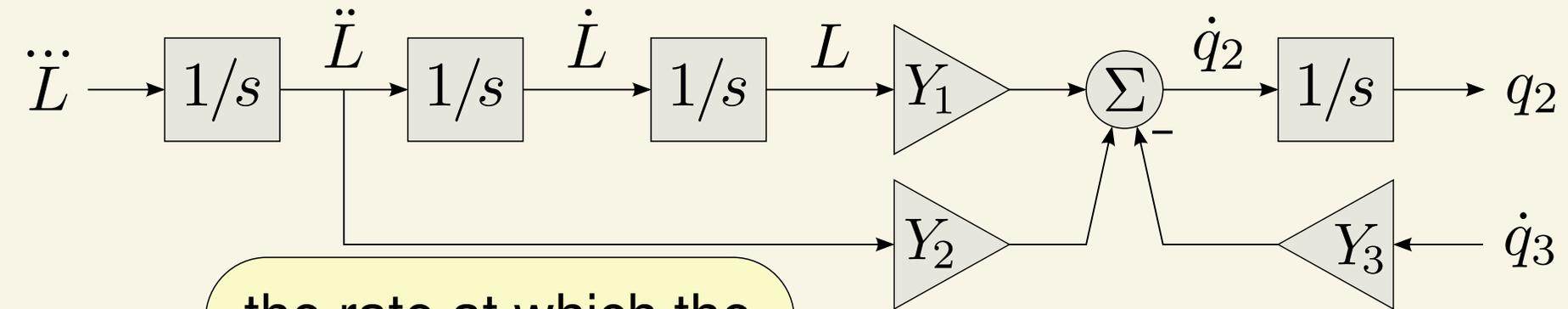
Y_1 and
properties

the rate at which the robot begins to fall, treating it as a single rigid body

two easily measured physical

- the natural time constant of toppling, and
- the linear velocity gain of \dot{q}_2

The Simple Dynamics of Balancing



Y_1 and
property

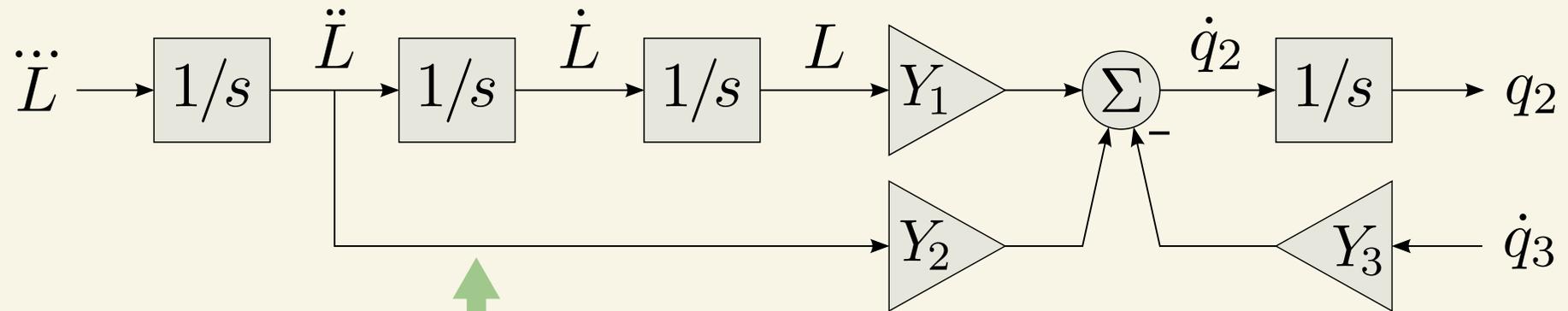
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- the natural time constant of toppling and
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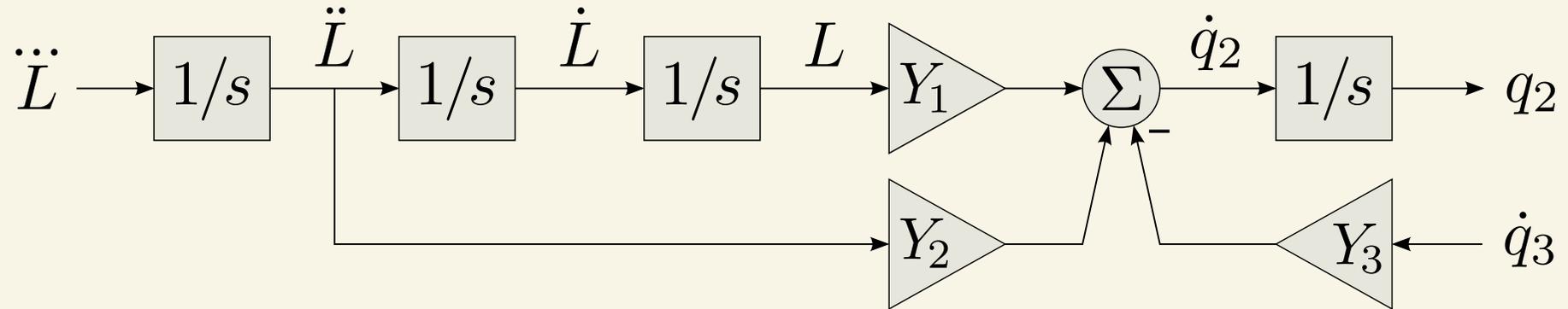
the step change in CoM velocity caused by a unit step change in \dot{q}_2

The Simple Dynamics of Balancing



In 2D, these signals are all scalars.
In 3D, they are 2D vectors.

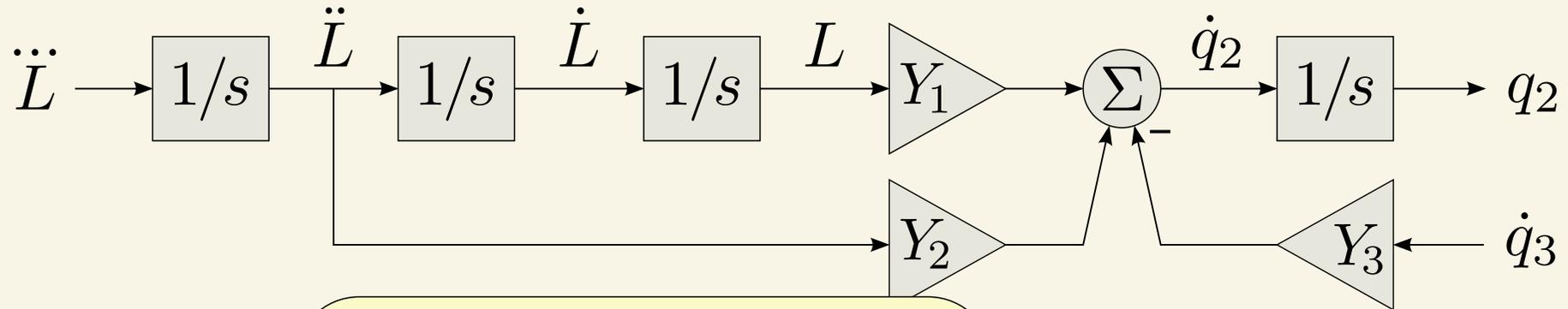
Simple Balance Control



High-performance balancing can be achieved using

- a simple control law closed around *this plant*
- and a simple *acausal filter* that implements *leaning in anticipation* of the balance disturbances that will be caused by commanded future motions.

Simple Balance Control



High-performance

a first-order low-pass filter running *backwards* in time from the future to the present

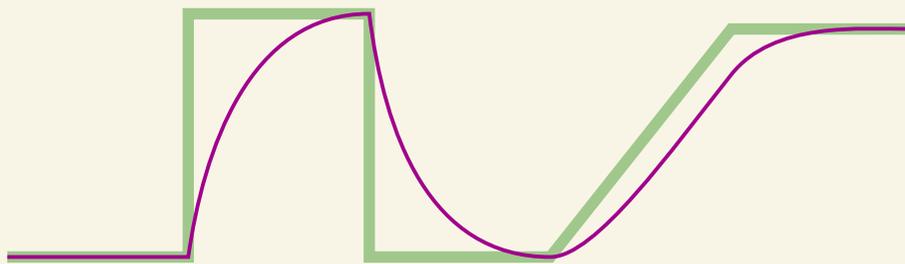
achieved using

- a simple *and this plant*
- and a simple *acausal filter* that implements *leaning in anticipation* of the balance disturbances that will be caused by commanded future motions.

The Acausal Filter

a first-order low-pass filter running backwards in time to cancel the non-minimum-phase response of the robot

normal low-pass filter



transfer function

$$\frac{1}{1 + T_c s}$$

natural time constant of toppling

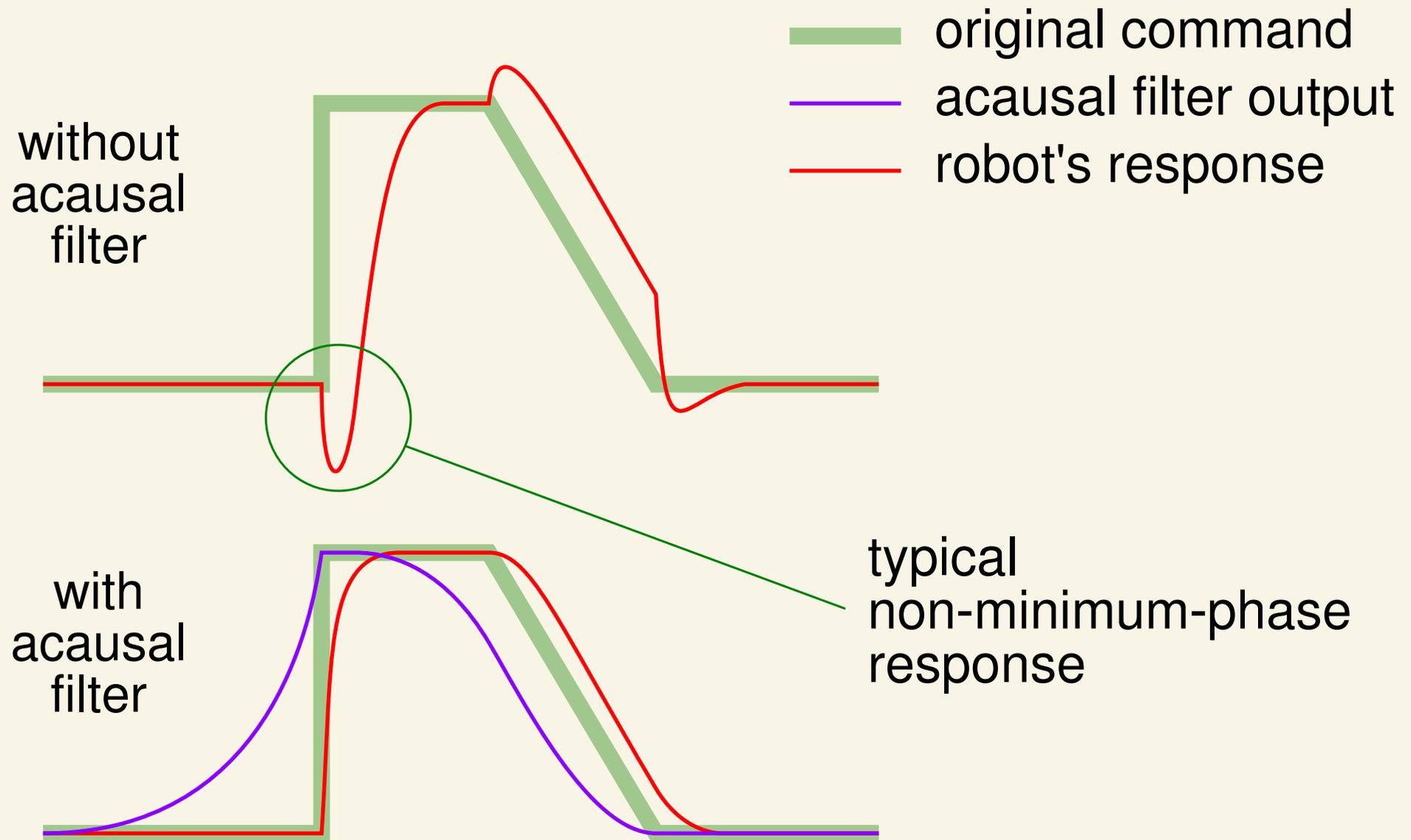
acausal low-pass filter



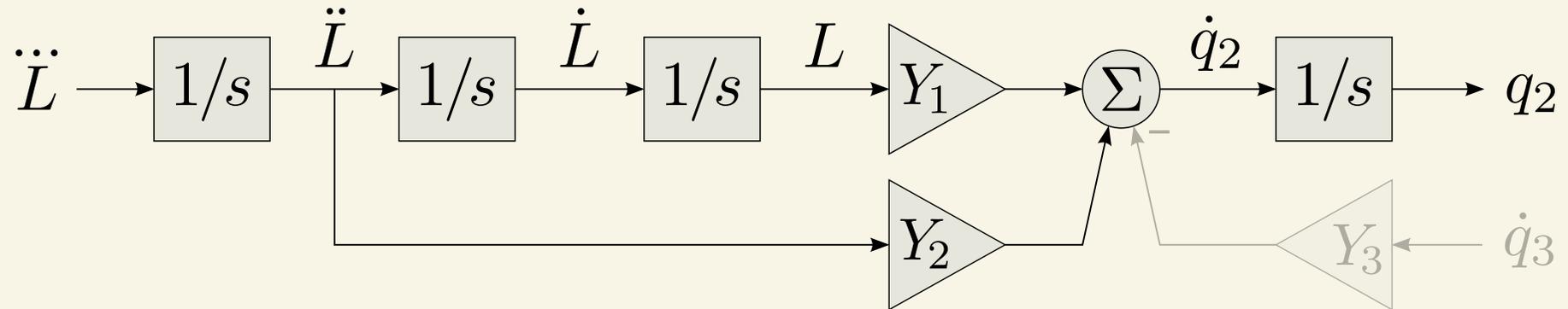
$$\frac{1}{1 - T_c s}$$

unstable pole in forward time

The Acausal Filter

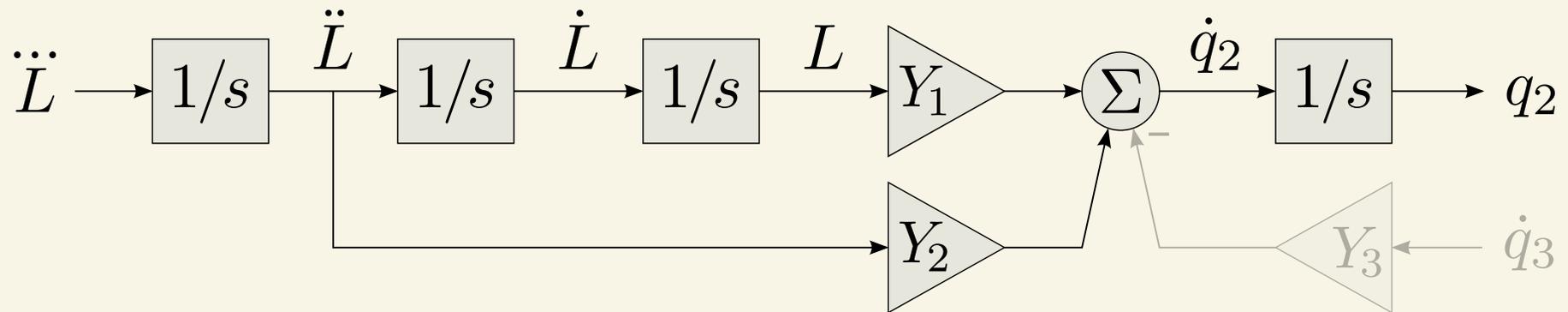


Balance Control Law



$$\ddot{L} = k_{dd}\ddot{L} + k_d\dot{L} + k_L L + k_q(q_2 - AF(u))$$

Balance Control Law



$$\ddot{L} = \underline{k_{dd}} \ddot{L} + \underline{k_d} \dot{L} + \underline{k_L} L + \underline{k_q} (q_2 - AF(u))$$

feedback gains

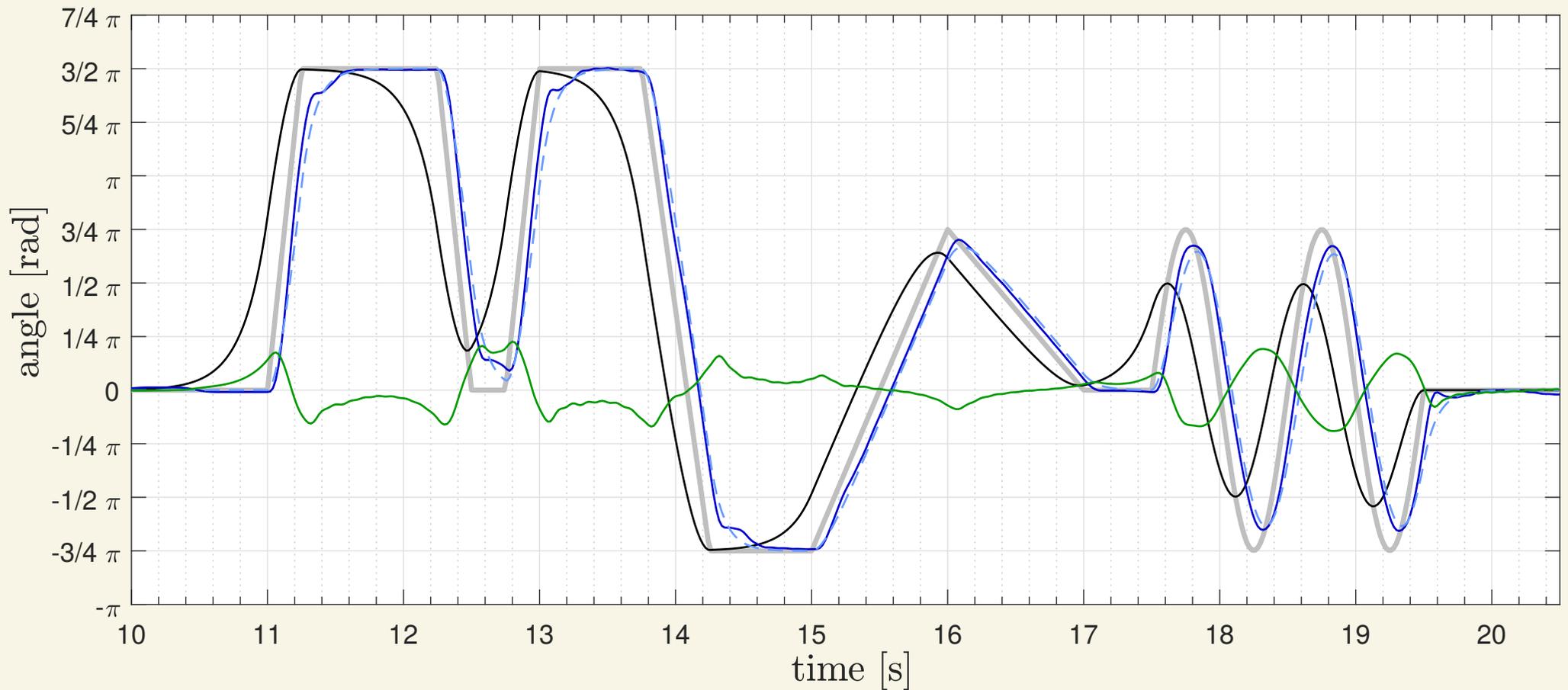
acausal filter

feedforward gains

$$u = \underline{q_{2c}} + \underline{\alpha_1} \dot{q}_{2c} + \underline{\alpha_2} \ddot{q}_{2c}$$

command signal and derivatives

Experimental Results (Tippy balancing using crossbar)



- q_c
- q_f
- q_2
- - - q_t
- $10q_1$

actual response q_2 very closely follows
theoretical response q_t

Website: <http://royfeatherstone.org/skippy>

Acknowledgements: the video of Tippy and the experimental results were produced by members of the Skippy Team

THE END